Extraction of SIFs and crack tip detection for curved cracks using digital images

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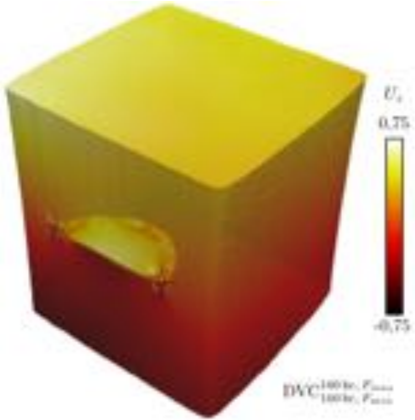
Introduction

SIF measurements using full-field measurements have shown its great potential:

- use of rich data field
- not limited to pure mode I
- can be extended to 3D cracks

The main difficulties are:

- crack tip position
- usually small displacement amplitudes
- influence of noise



State of the art

- Projection based techniques

  idea: LS projection of the measured displacement onto an appropriate basis

  McNeill *et al.*EFM, 1987. Abanto-Bueno and Lambros, EM, 2005. Yoneyama *et al.*, Strain, 2006. Hamam *et al.*, Strain, 2006. Limodin Acta Mat., 2009.

- Conservation law based techniques

  idea: Invariant domain integral computed using the measured displacement

  Réthoré *et al.*, IJF, 2005. Rannou *et al.*, CMAME, 2009. Pop *et al.*, IJF, 2011.

- Optimization of noise robustness

  idea: Recast the extraction as a minimization problem under extraction constraints

The *integrated* approach W-DIC

The *BEST* method is the integrated approach by S. Roux and F. Hild.
The optical flow

$$f(\mathbf{X_p}) = g(\mathbf{X_p} + \mathbf{u}_w(\mathbf{X_p})),$$

is solved using crack tip kinematics:

$$u_w = \sum_{j=I,II} \sum_{n=n_m \ldots n_M} a_j^n \bar{\phi}_j^n(z) \, .$$

Roux and Hild, IJF, 2006.

Williams' series

For a linear elastic solid with a semi infinite crack, the solution can be decomposed over Williams' series:

$$\bar{\phi}_I^n(r,\theta) = r^{n/2}\left(\kappa e^{in\theta/2} - \frac{n}{2}e^{i(4-n)\theta/2} + (\frac{n}{2} + (-1)^n)e^{-in\theta/2}\right)$$

$$\bar{\phi}_{II}^n(r,\theta) = ir^{n/2}\left(\kappa e^{in\theta/2} + \frac{n}{2}e^{i(4-n)\theta/2} - (\frac{n}{2} - (-1)^n)e^{-in\theta/2}\right)$$

- $n > 2$ sub-singular terms
- $n = 2$ rotation and T-stress
- $n = 1$ usual terms
- $n = 0$ translations

Williams M. On the stress distribution at the base of a stationary crack. *ASME Journal Applied Mechanics* 1957; **24**:109–114.

Crack tip positioning

- From the previous expressions, we have the following recurrence formula:

$$\frac{\partial \bar{\phi}_j^n}{\partial x} = \frac{n}{2} \bar{\phi}_j^{n-2}$$

- If the crack tip is mis-positioned by a small distance d along its axis

$$u = \sum_{n \geq 0, j} K_j^n \bar{\phi}_j^n(z) = \sum_{n,j} \tilde{K}_j^n \bar{\phi}_j^n(z+d)$$

- Then using a Taylor expansion, and identifying the different terms, we have:

$$d = \frac{2\tilde{K}_I^{-1}}{\tilde{K}_I^1}$$

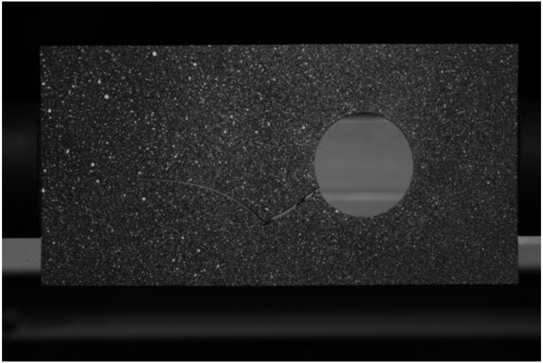
S. Roux *et al.*. IJF 2006.

Motivations

140 mm×70 mm×10 mm, hole 30 mm, $E = 5GPa$, $\nu = 0.32$

4872×3248 pixels images, 32.5 μm/pixel

Motivations

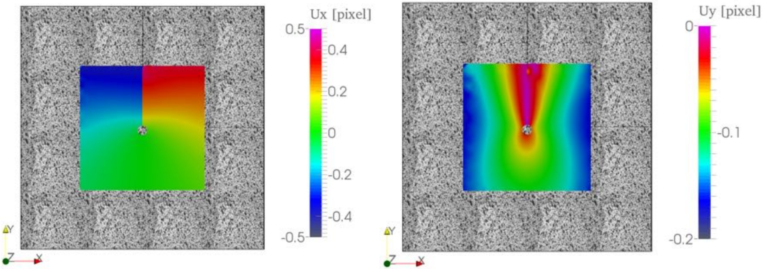


Williams' series only valid for a straight crack

$\longrightarrow$ extraction domain $\searrow$ $\longrightarrow$ uncertainty $\nearrow$

Virtual test

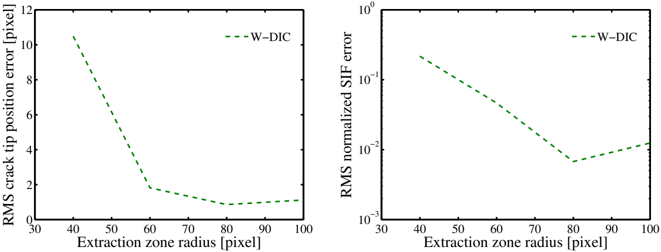
A serie of virtual images is generated with a moving crack tip at a constant K_I



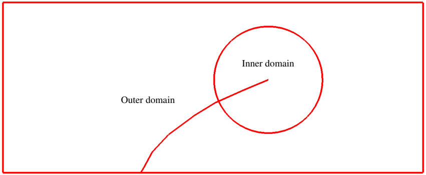
1024×1024 pixels

Virtual test

W-DIC results



Proposed methodology



- Inner domain:
  - SIF estimates
  - crack tip position
  - small size compared to the radius of curvature
- Outer domain:
  - arbitrary kinematics
  - transfert usefull data to the inner domain

Proposed methodology

- Inner domain: DIC based on Williams' series

$$u_w = \sum_{j=I,II} \sum_{n=n_m \dots n_M} a_j^n \bar{\phi}_j^n(z) \; ; \; f(\mathbf{X_p}) = g(\mathbf{X_p} + \mathbf{u}_w(\mathbf{X_p}))$$

- Outer domain: FEM-DIC + elastic regularization

$$u_{FE}(\mathbf{X}) = \sum_{k=1 \dots 2N} u_k N_k(\mathbf{X}) \; ; \; f(\mathbf{X_p}) = g(\mathbf{X_p} + \mathbf{u}_w(\mathbf{X_p})) \; ; \; \{\bar{\mathbf{F}}_{\mathbf{int}}\}^T \{\bar{\mathbf{F}}_{\mathbf{int}}\} = 0$$

A coupling between the two descriptions is required:

- overlapping domain with energy partition
  - Réthoré J. *et al.*, *IJNME* 2010, *CR Méca* 2010.
- no overlapping with kinematic coupling (Mortar method)
  - Passieux JC. *et al.*, *IJNME* 2011, *Comp. Meth.*, 2013.

Proposed methodology

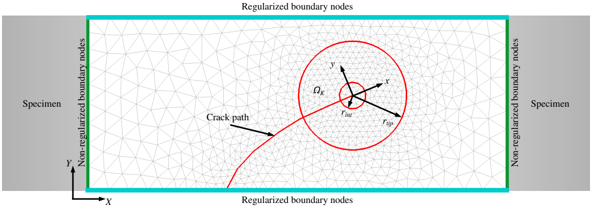
DIC + elastic regularization

1. Optical flow

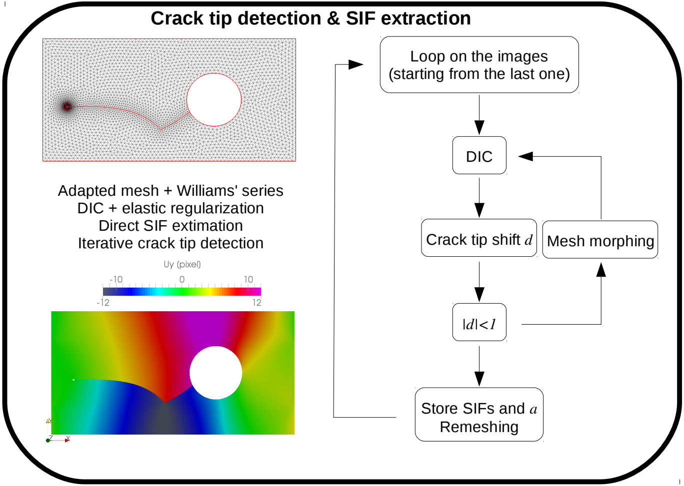
$$f(\mathbf{X_p}) = g(\mathbf{X_p} + \mathbf{u}(\mathbf{X_p})) \rightarrow [\mathbf{M}]\ \{\mathrm{d}\mathbf{U}\} = \{\mathbf{b}\}.$$

2. Equilibrium gap (**P** selects the regularized nodes)

$$\{\bar{\mathbf{F}}_{\mathbf{int}}\}^T \{\bar{\mathbf{F}}_{\mathbf{int}}\} = 0 \rightarrow [\mathbf{K}]^T [\mathbf{P}]^T [\mathbf{P}] [\mathbf{K}] \{\mathbf{U}\} = 0$$

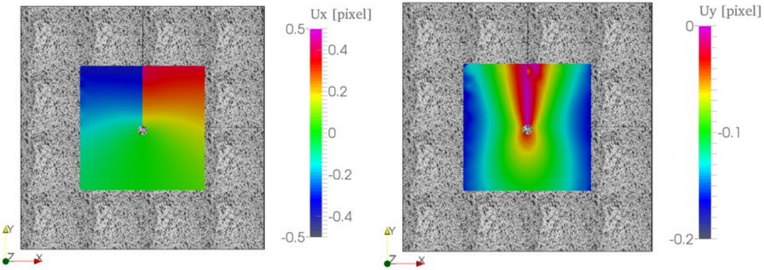


Outline


Crack tip detection & SIF extraction
Adapted mesh + Williams' series
DIC + elastic regularization
Direct SIF extimation
Iterative crack tip detection
Uy (pixel)
-10
0
10
-12
12
Loop on the images
(starting from the last one)
DIC
Crack tip shift d
Mesh morphing
$|d|<1$
Store SIFs and a
Remeshing

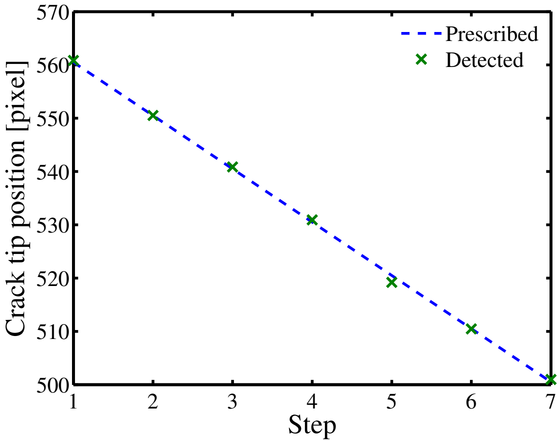
Validation

A serie of virtual images is generated with a moving crack tip at a constant K_I

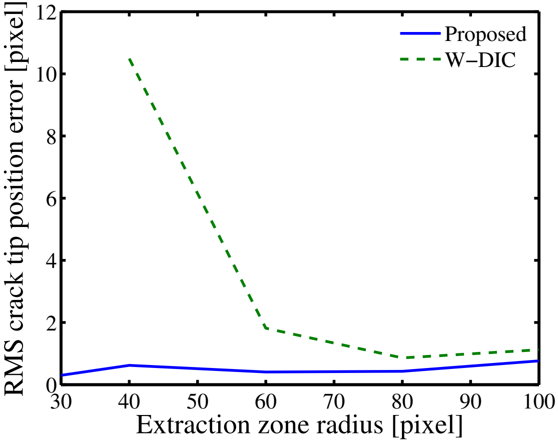


1024×1024 pixels

Validation

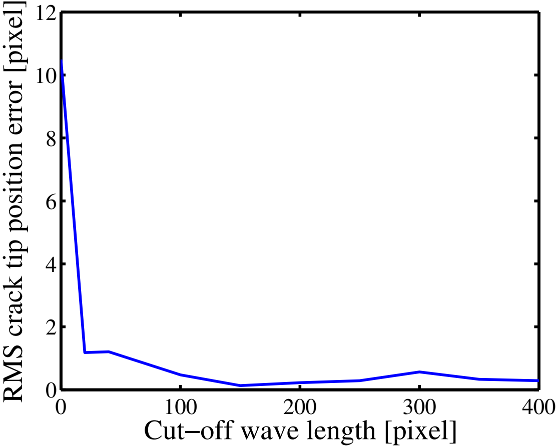

570
560
550
540
530
520
510
500
Crack tip position [pixel]
1
2
3
4
5
6
7
Step
Prescribed
Detected

Validation



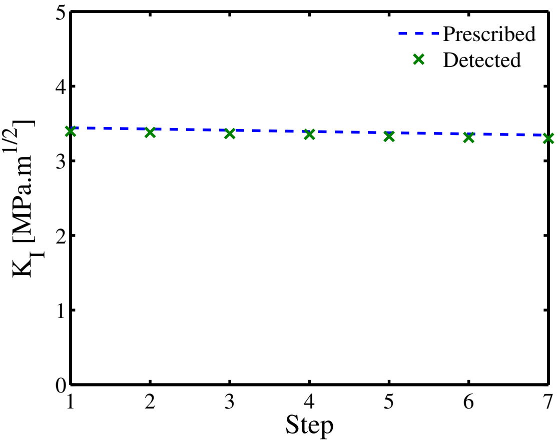
$\ell_c = 400$ pixels

Validation

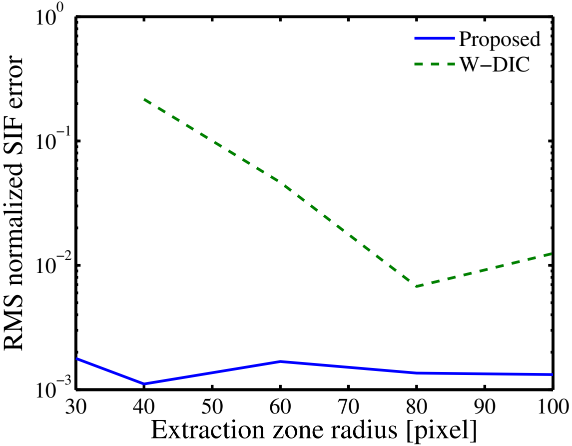


r_{tip} =40 pixels

Validation

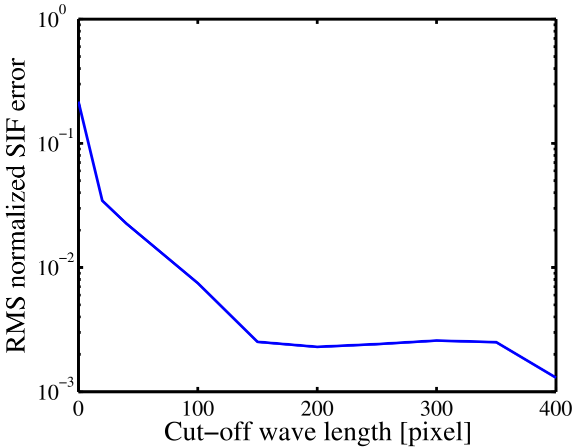


Validation



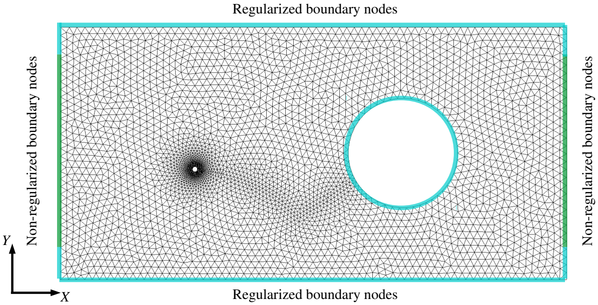
$\ell_c = 400$ pixels

Validation



r_{tip} =40 pixels

Numerical setup

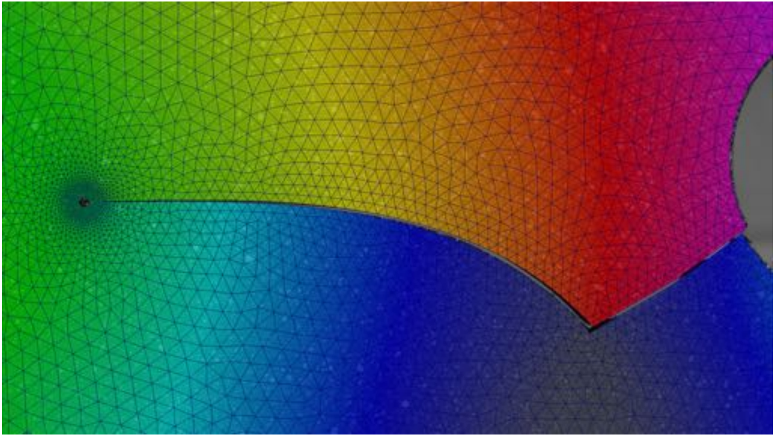


$h_{outer} = 64$ pixels, $r_{tip} = 64$ pixels, $h_{tip} = 8$ pixels, $\ell_c = 256$ pixels

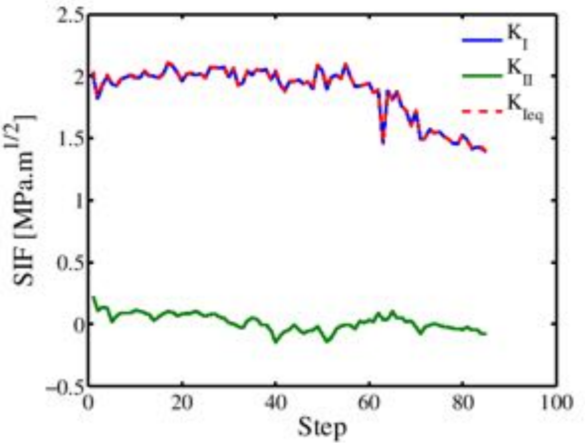
$\sigma(\mathbf{n}) = 0$ along the crack faces

Results

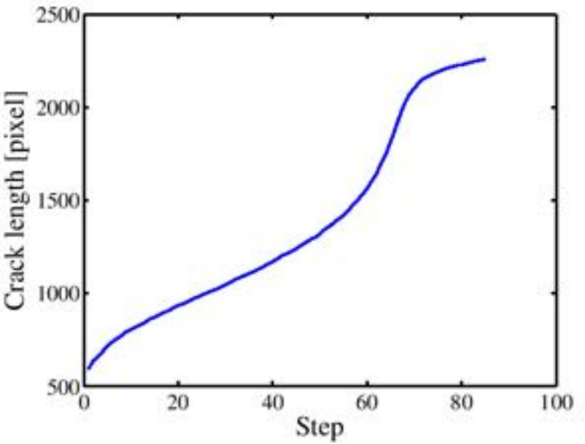
Results



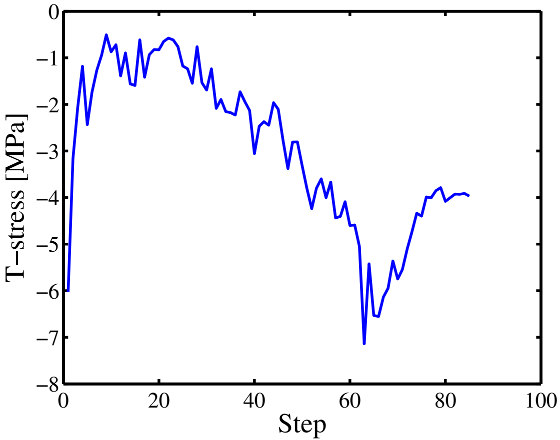
Results



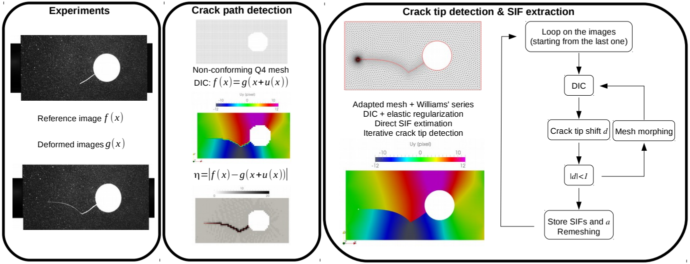
Results


2500
2000
1500
1000
500
0
20
40
60
80
100
Crack length [pixel]
Step

Results



Summary


Experiments
Reference image $f(x)$
Deformed images $g(x)$
Crack path detection
Non-conforming Q4 mesh
DIC: $f(x)=g(x+u(x))$
$\eta=|f(x)-g(x+u(x))|$
Crack tip detection & SIF extraction
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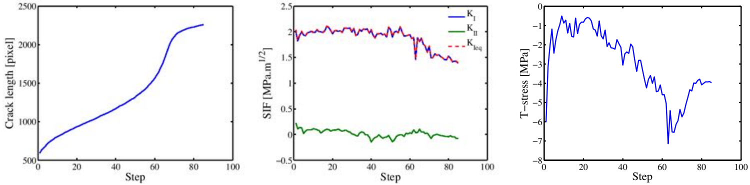
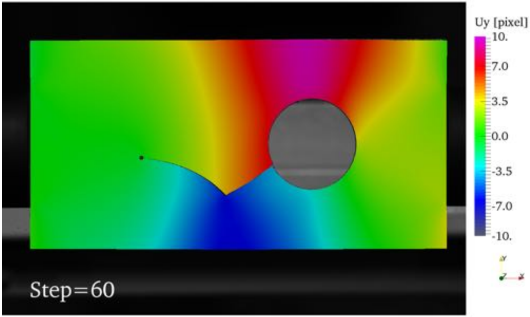
Perspectives: criteria...





Mesh morphing

Elastic simulation with appropriate BCs