The Thick Level Set (TLS) damage model for quasi-brittle fracture: state of the art

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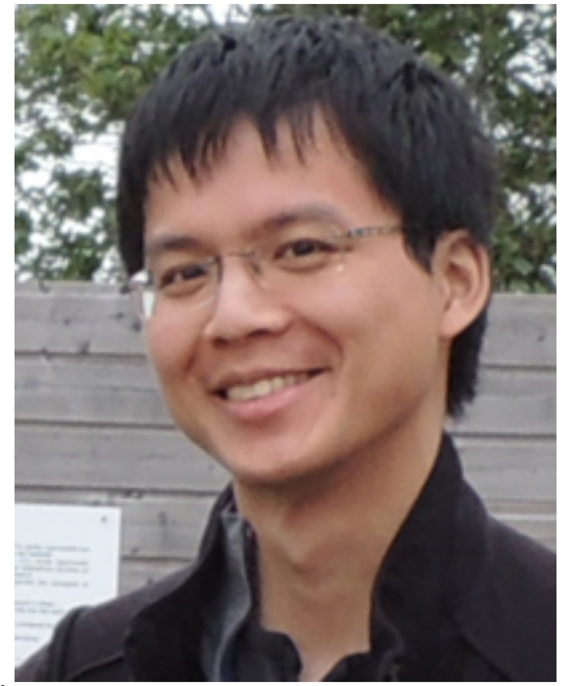
- Fabien Cazes, Paul-Emile Bernard (Former Post-Docs)
- T. Gorris, A.E. Selke, **A. Parilla-Gomez**,   (Former Phd students)






Outline

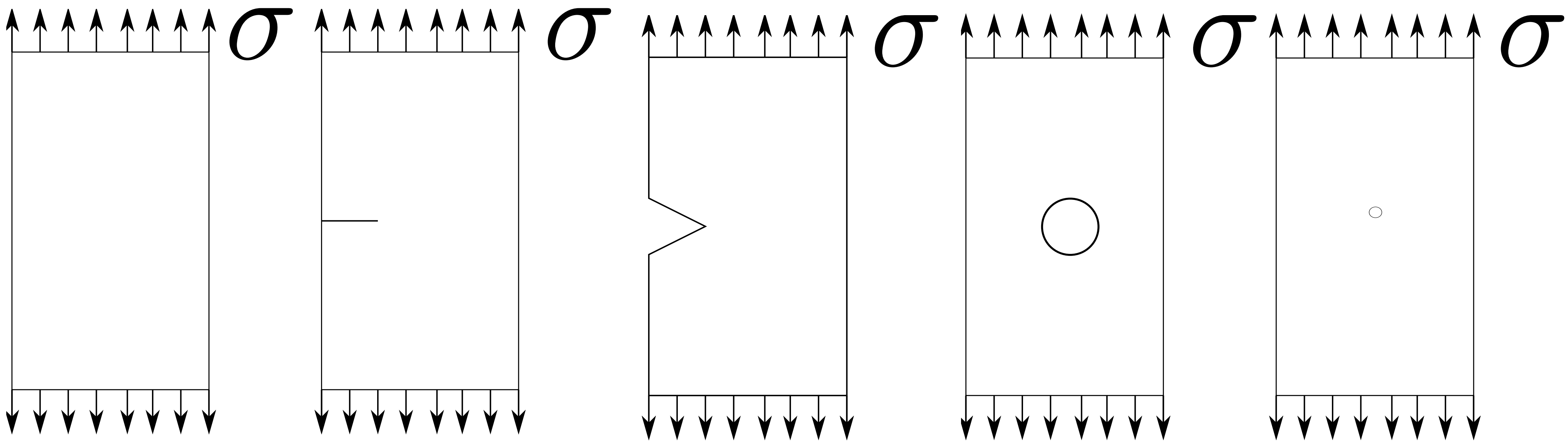
- Foreword : Material vs Structural testing,
- Discontinuity models with rising complexity
- TLS basics: transition from damage to fracture
- TLS generalizes Cohesize Zone Models
- Conclusion and Future work.

Foreword (I)

- No scale separation when it comes to strain localization and fracture.
- Simple tests are already structural tests, there exist no pure material test because failure is not homogeneous.
- Material modeling must involve a length scale. It interacts with the structure whatever its size.
- Models and simulation is needed to get informations from tests.
- The notion of local failure criteria (stress, strain, combination) is ill-posed.

Arguments on : The need for a length

Limit stress in quasi-static



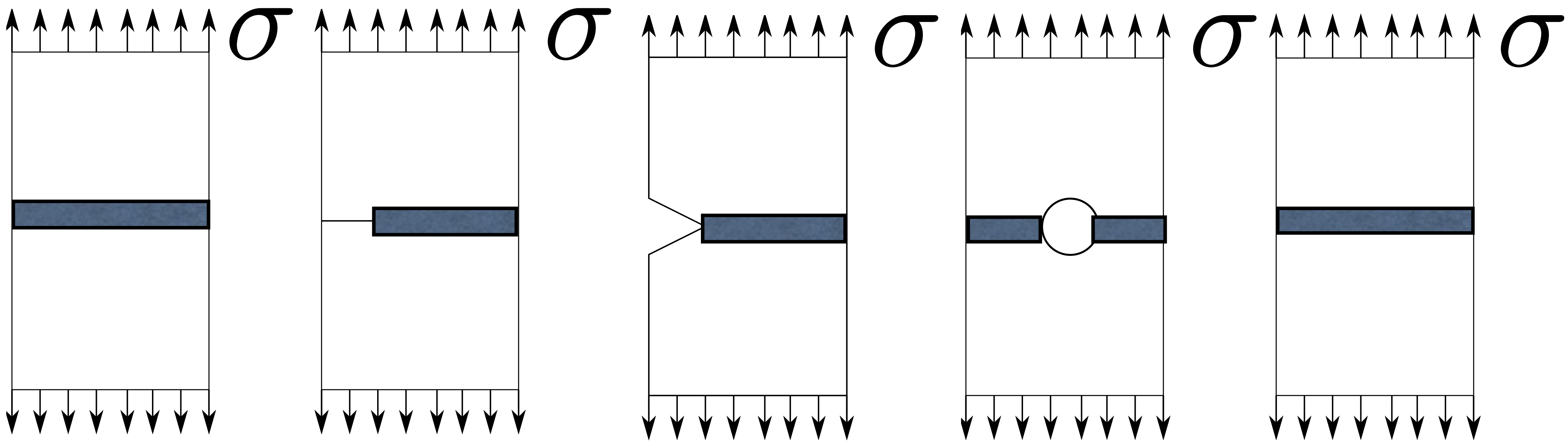
$\sigma \leq \sigma_c$ | $G \leq G_c$ | ? | $\sigma \leq \sigma_c/3$ | $\sigma \leq \sigma_c$

Strength | Toughness

$$l_c \sim \frac{(EG_c)}{(\sigma_c)^2}$$

The notion of local failure criteria is ill-posed.

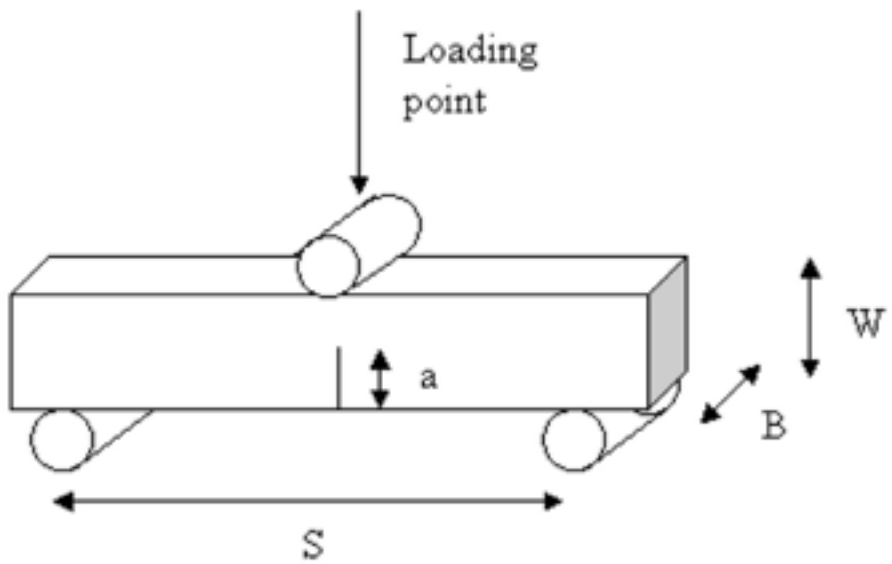
Limit stress in quasi-static

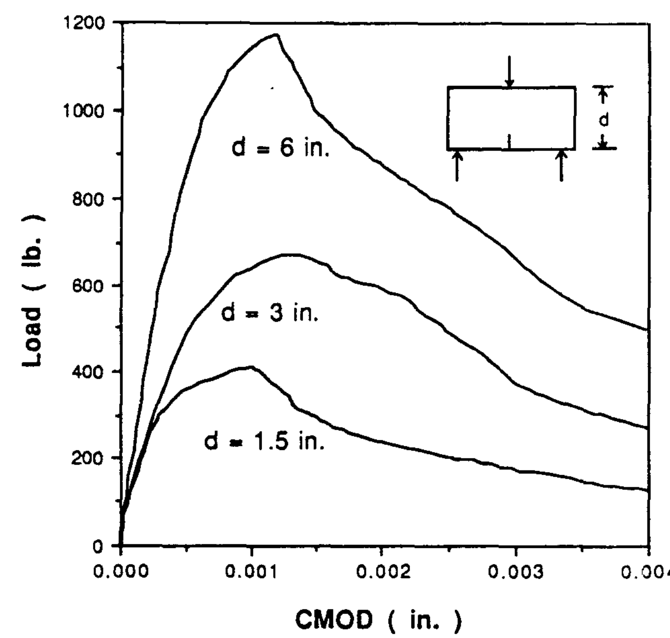


$\sigma \leq \sigma_c$ $\quad$ $G \leq G_c$ $\quad$? $\quad$ $\sigma \leq \sigma_c/3$ $\quad$ $\sigma \leq \sigma_c$

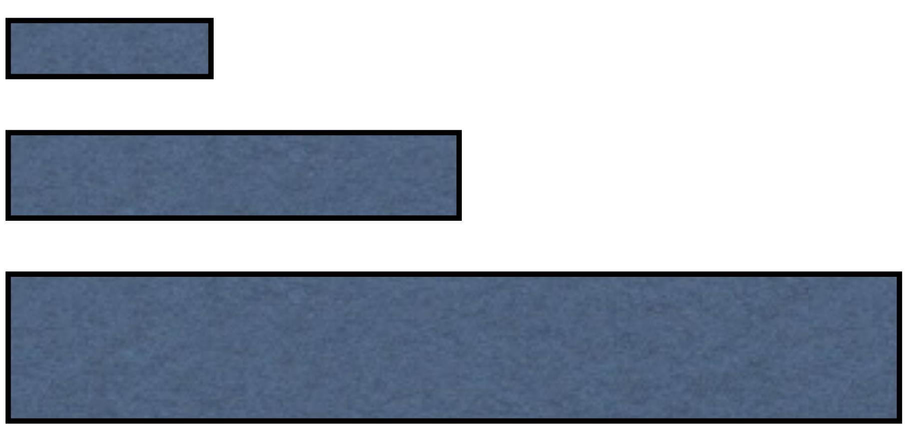
$$l_c \sim \frac{(EG_c)}{(\sigma_c)^2}$$

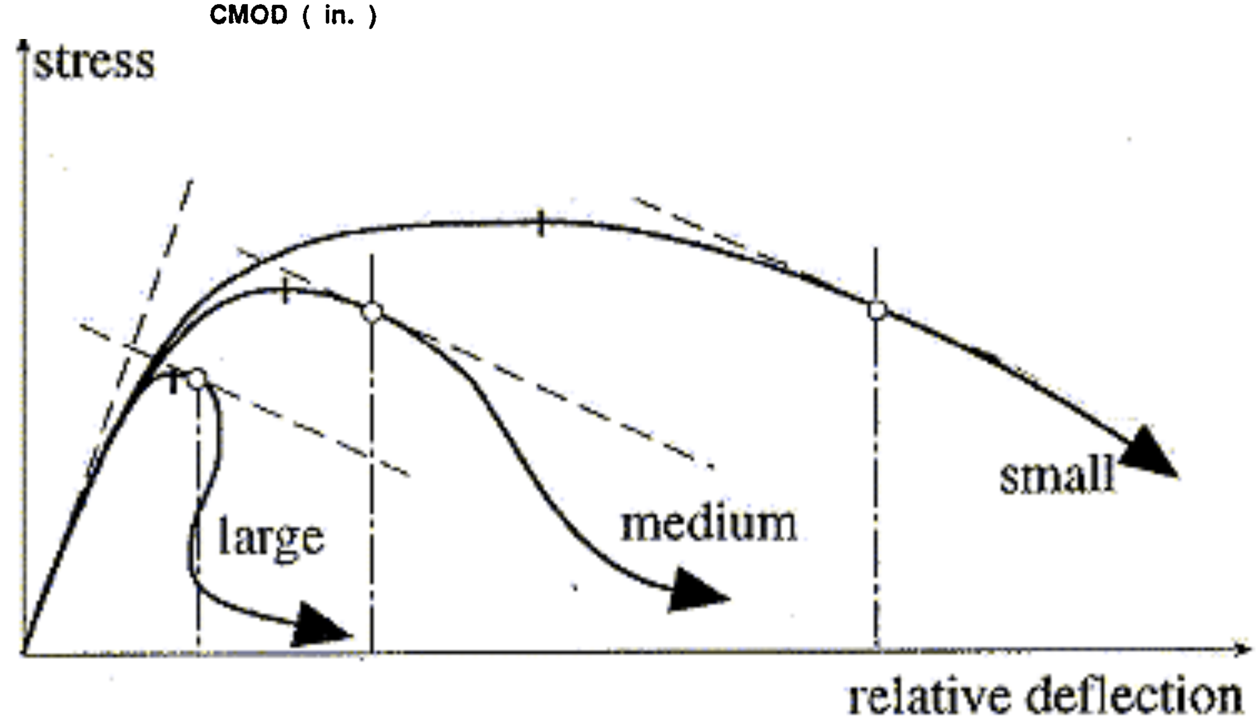
The notion of size effect
(explained on a Three Point Bending Test)





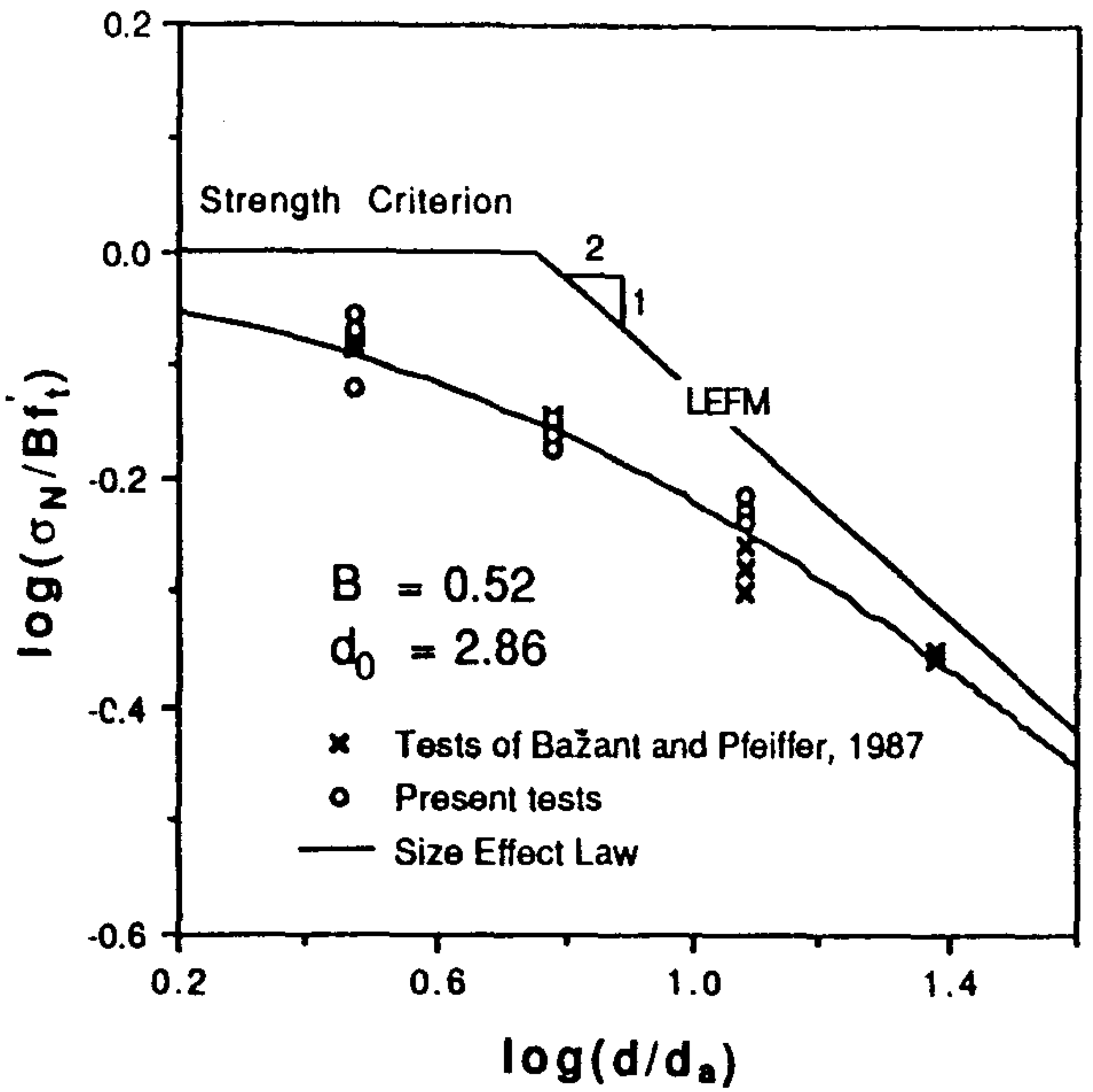
Bazant, Xu 1991





And the notion of quasi-brittle material

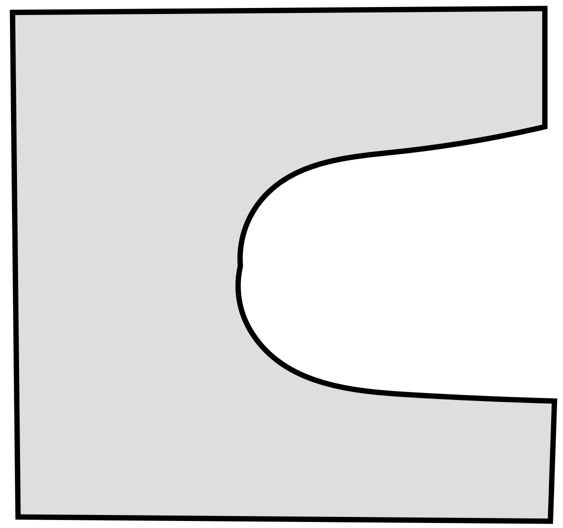
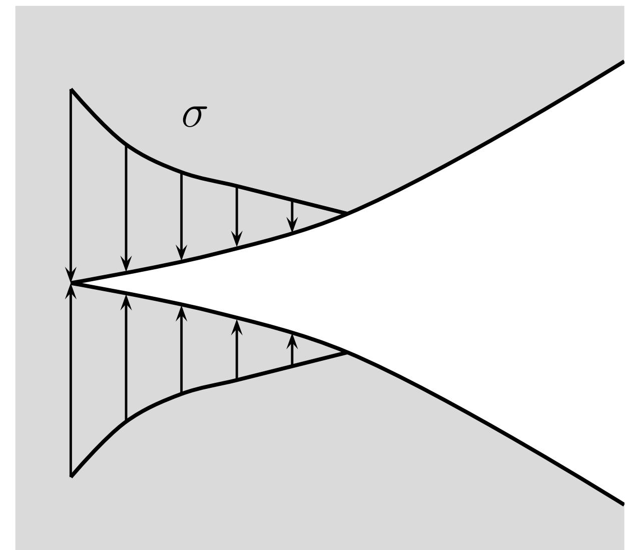
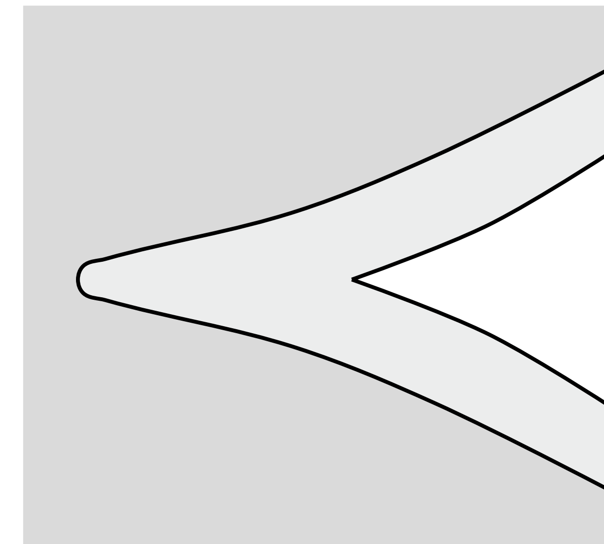
Bazant size effect law



Foreword (II)

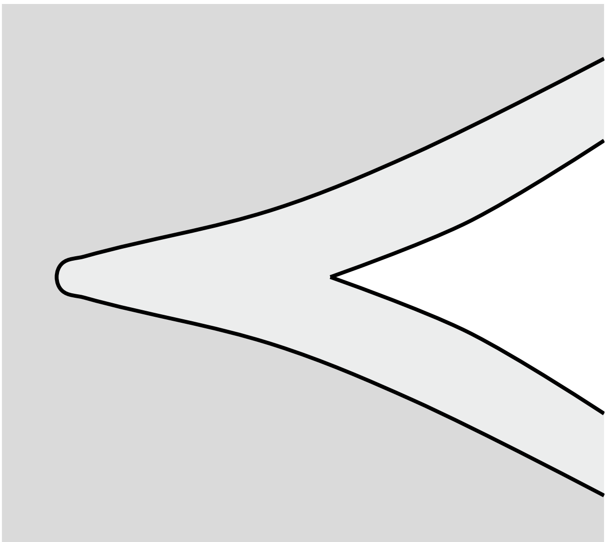
- Pavement degradation : Damage, cracking, branching, debonding, permanent deformation : the need for a get together model.
- This talk : (a) link between damage and fracture, (b) link between debonding-damage-cracking model

Discontinuity models with rising complexity

Griffith	Cohesive	Damage
		

Length scale	No	Yes	Yes (if non-local)
Initiation	No	Yes	Yes
Branching or complex patterns	No	No	Yes

Does this model exist?



- Growing discontinuity (displacement jumps)
- Localization zone surrounded by a local zone
- No material euthanasia (smooth transition to fracture)
- Continuum mechanics based

Not really

- Damage based gradient model are non-local everywhere and at all times
- Appearance of displacements jumps is not part of the damage based gradient models
- Morphing (G. Lubineau) may be used to couple local and non-local models
- Peridynamics tend to create discontinuity but is not continuum based

In fact Yes, it was the motivation for the Thick Level Set approach to fracture

$$\|\nabla d\| \leq \frac{f(d)}{l_c} \qquad \text{TLS}$$

$$\Delta d = \frac{g(d,\epsilon)}{l_c} \qquad \text{Damage gradient}$$

- Inequality -> non-intrusive non-locality
- First order gradient (Hamilton-Jacobi)
- No boundary conditions for d (just intial)
- g depends on the damage model
- f independent of local damage equation.

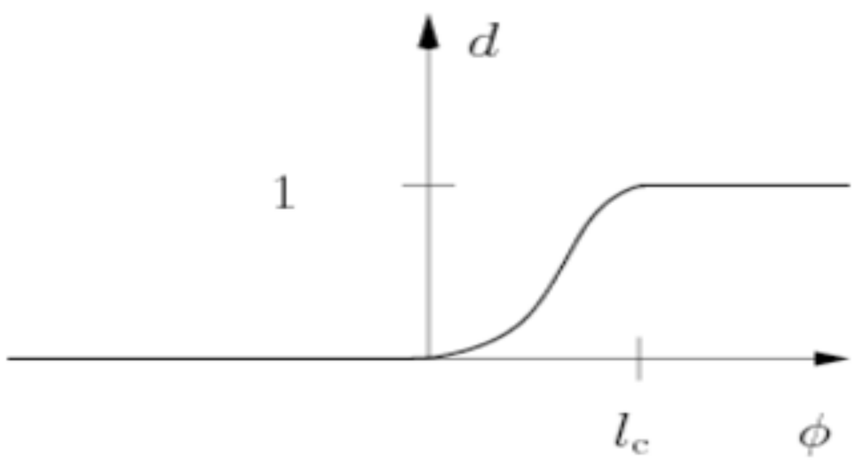
Geometrical nature of the TLS

TLS = CDM et $\|\nabla d\| \leq f(d) \text{ on } \Omega$

Identical to (Eikonal inequality)

$$\begin{cases} \|\nabla \phi\| \leq 1 \\ d = d(\phi) \end{cases}$$

$$f(d) = d'(\phi(d))$$

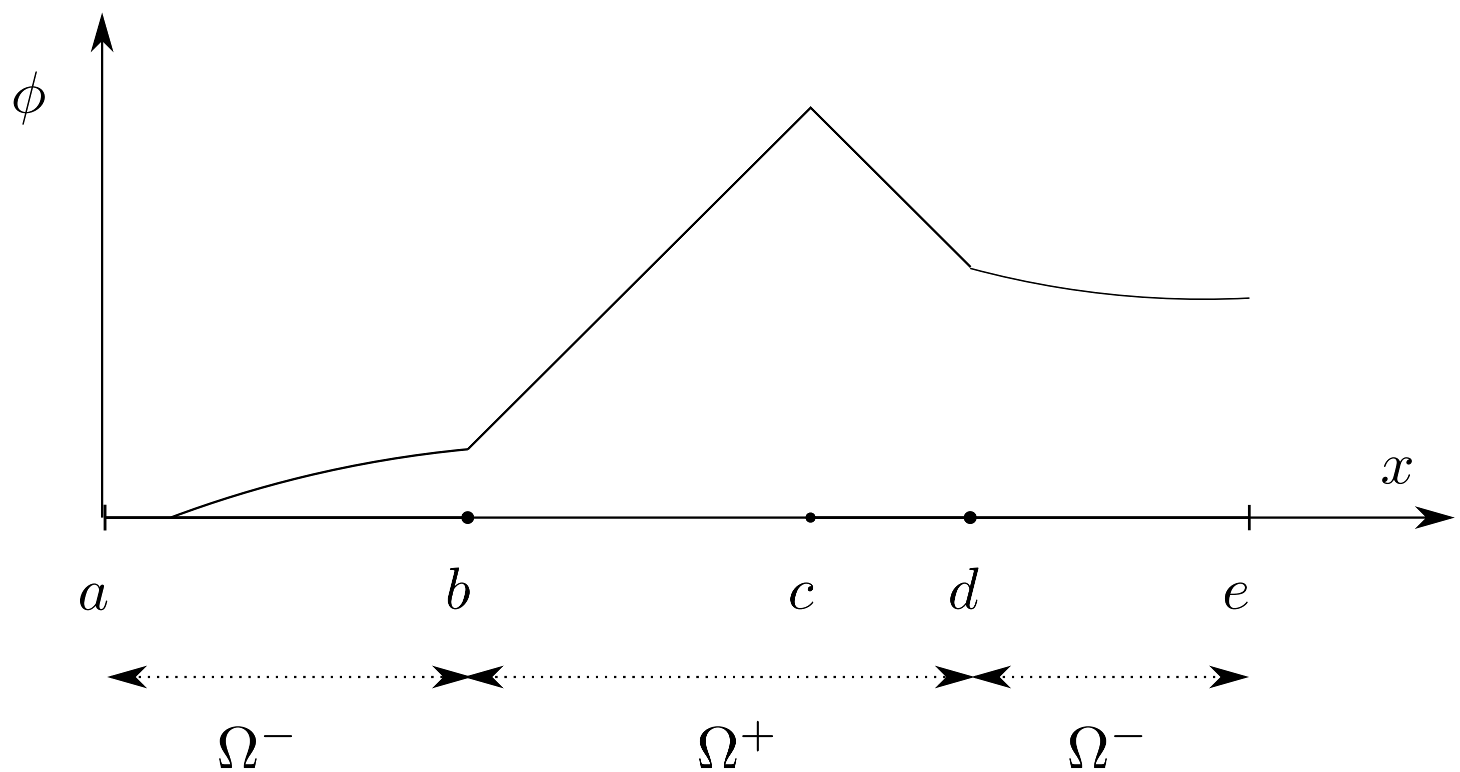


Crack is located automatically (iso-lc)

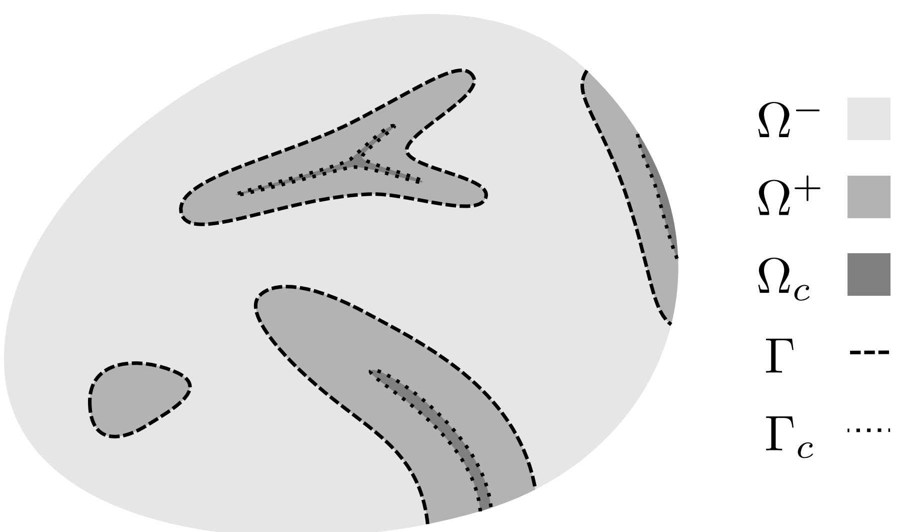
$$\|\nabla\phi(\boldsymbol{x})\| < 1 \quad \Rightarrow \quad \text{Local constitutive model at } \boldsymbol{x}$$

$$\|\nabla\phi(\boldsymbol{x})\| = 1 \quad \Rightarrow \quad \text{Non-Local constitutive model at } \boldsymbol{x}$$

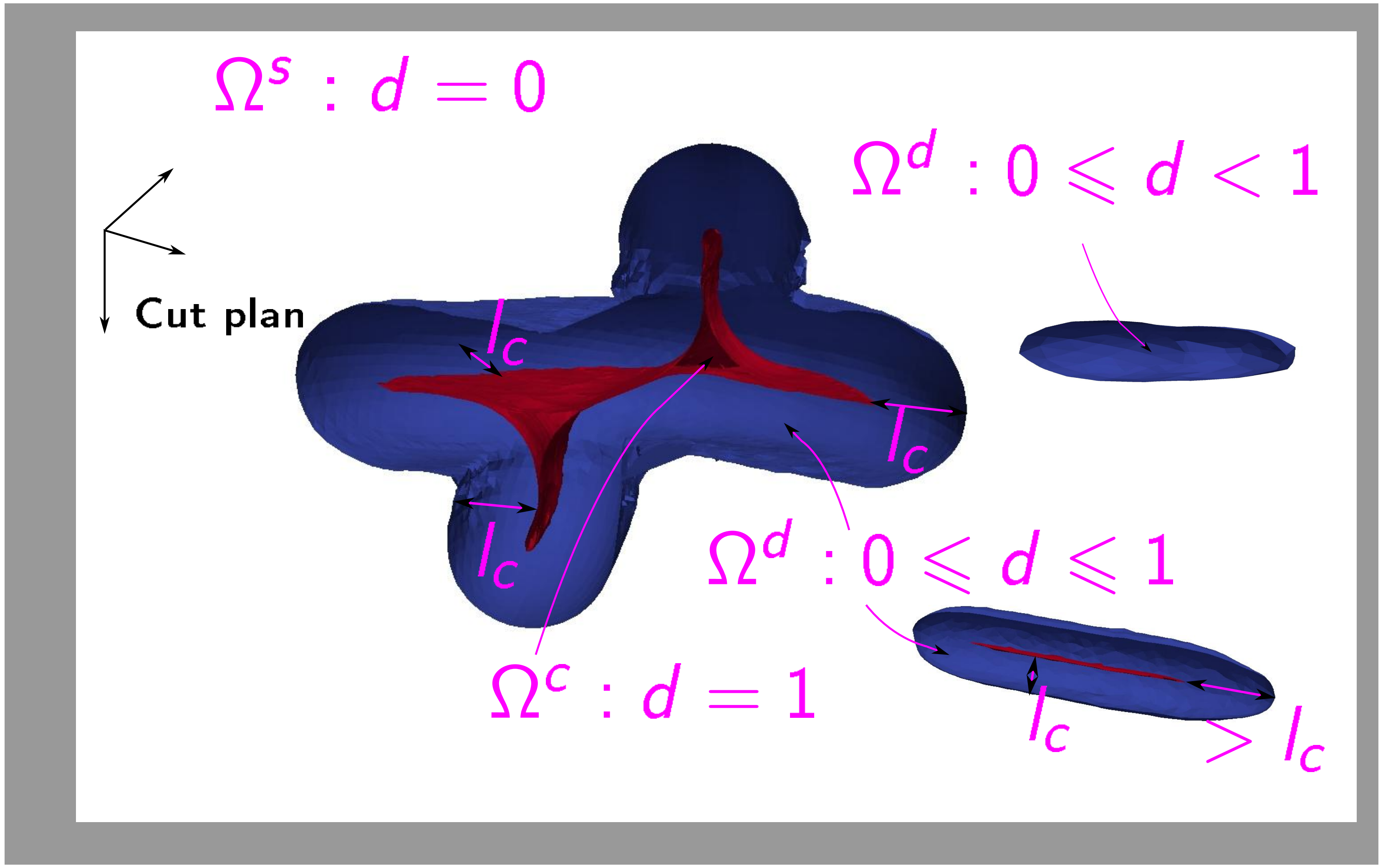
$$\|\nabla\phi(\boldsymbol{x})\| > 1 \qquad \text{forbidden}$$



1D

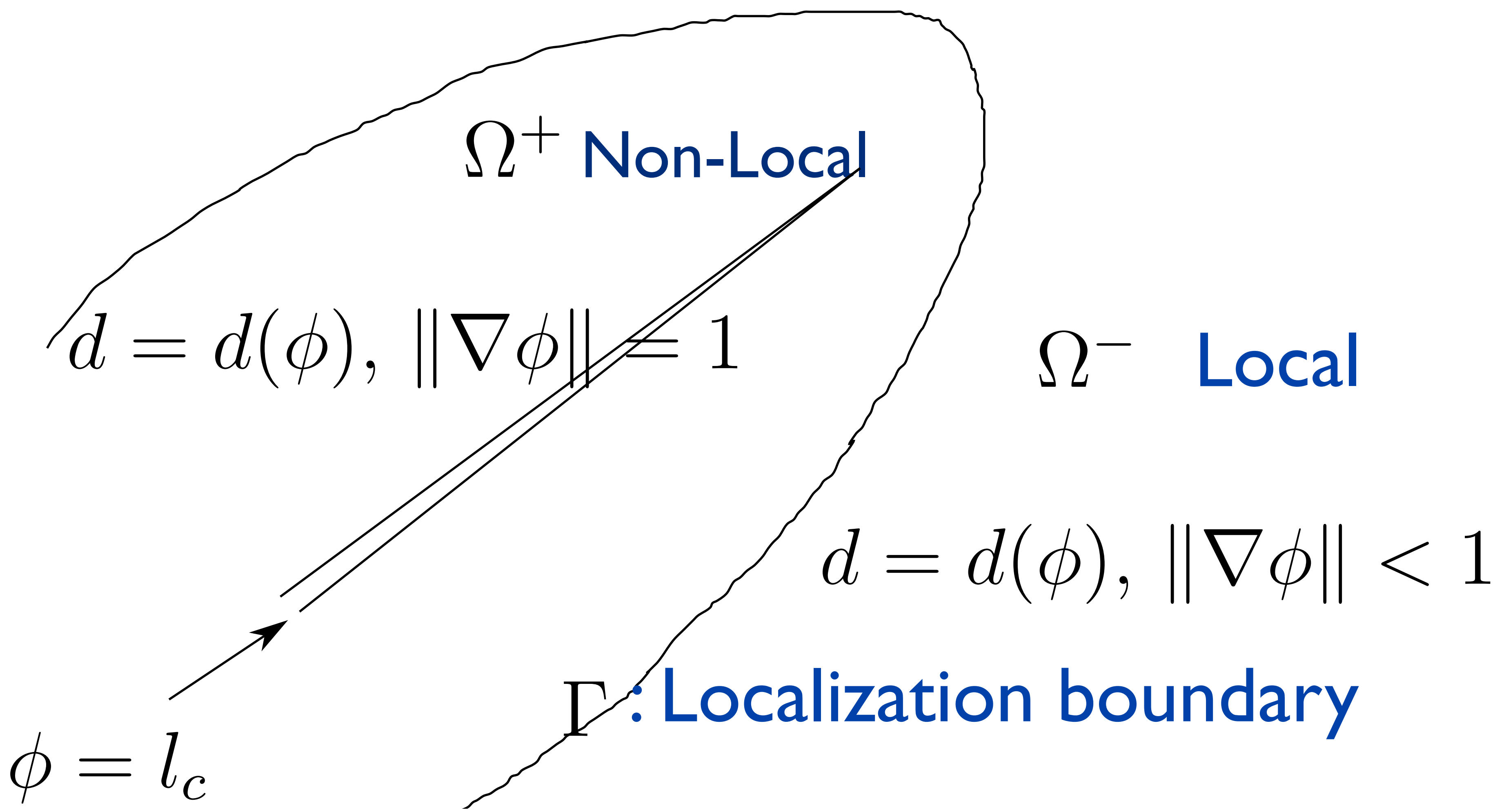


2D



3D

Local and non-local damage zones



The localization boundary evolves in order to preserve damage continuity (Hadamard condition)

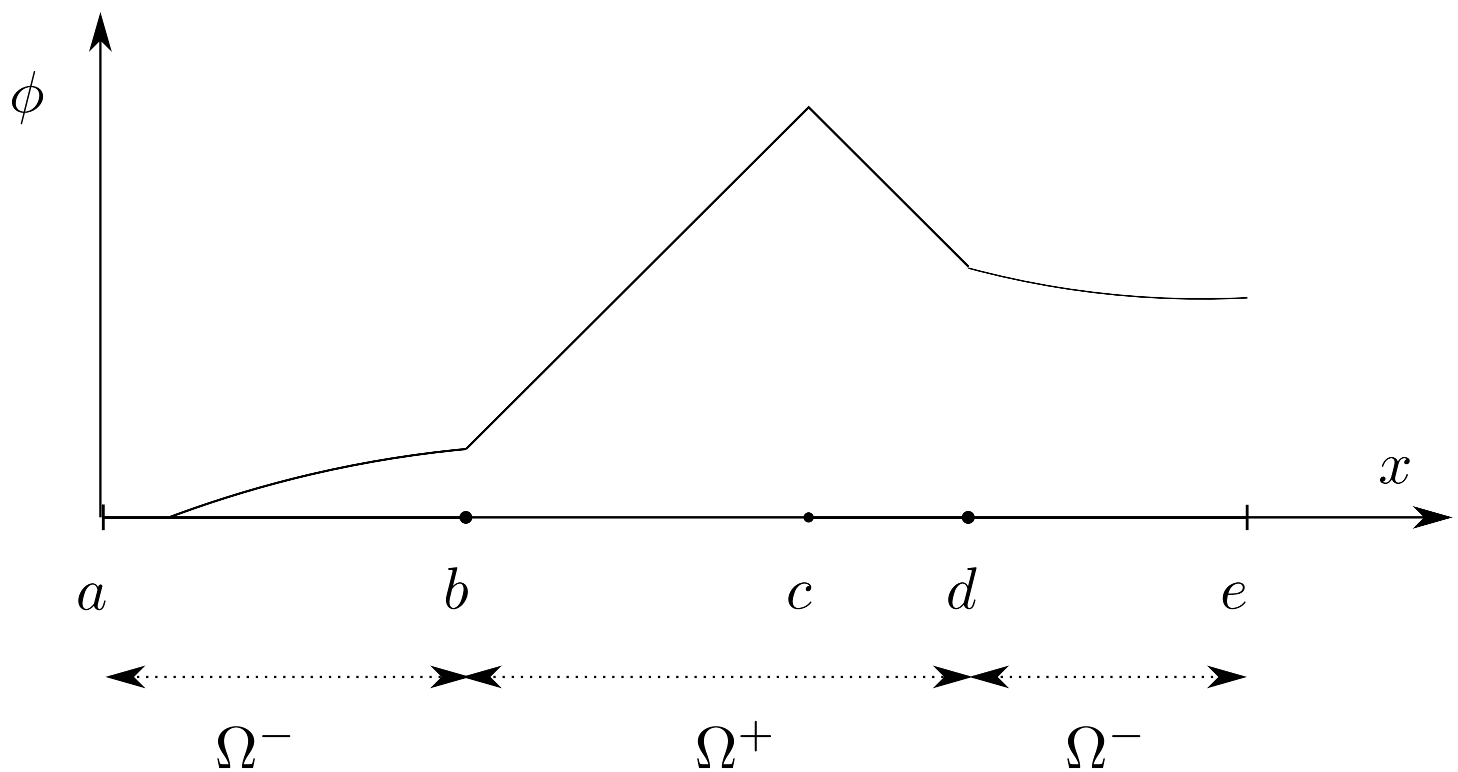
$$[d]_\Gamma = 0 \ \ \forall t \quad \Rightarrow \quad [\mathring{d}]_\Gamma = 0 \quad \Rightarrow \quad [\dot{d}] + \mathbf{v}_\Gamma \cdot [\nabla d]_\Gamma = 0$$

	LEFM	TLS Damage	Damage
Energy	$\int_{\Omega \backslash a} w(u) \, \mathrm{d}\Omega$	$\int_{\Omega} w(u, d(\phi))$	$\int_{\Omega} w(u, d)$
state equ.	$\sigma = \frac{\partial w}{\partial \epsilon(u)}$	$\sigma = \frac{\partial w}{\partial \epsilon(u)}$	$\sigma = \frac{\partial w}{\partial \epsilon(u)}$
state equ.	$G = -\frac{\partial W}{\partial a}$	$\overline{Y} =< Y >$	$Y = -\frac{\partial w}{\partial d}$
Dissipation	$G\dot{a}$	$\int_{\Omega} \overline{Y}\overline{\dot{d}} \, d\Omega$	$\int_{\Omega} Y\dot{d}$
evol. eq.	$\dot{a} = \frac{\partial \psi^*(G)}{\partial G}$	$\overline{\dot{d}} = \frac{\partial \psi^*(\overline{Y})}{\partial \overline{Y}}$	$\dot{d} = \frac{\partial \psi^*(Y)}{\partial Y}$

$$\|\nabla\phi(\boldsymbol{x})\| < 1 \quad \Rightarrow \quad \text{Local constitutive model at } \boldsymbol{x}$$

$$\|\nabla\phi(\boldsymbol{x})\| = 1 \quad \Rightarrow \quad \text{Non-Local constitutive model at } \boldsymbol{x}$$

$$\|\nabla\phi(\boldsymbol{x})\| > 1 \qquad \text{forbidden}$$

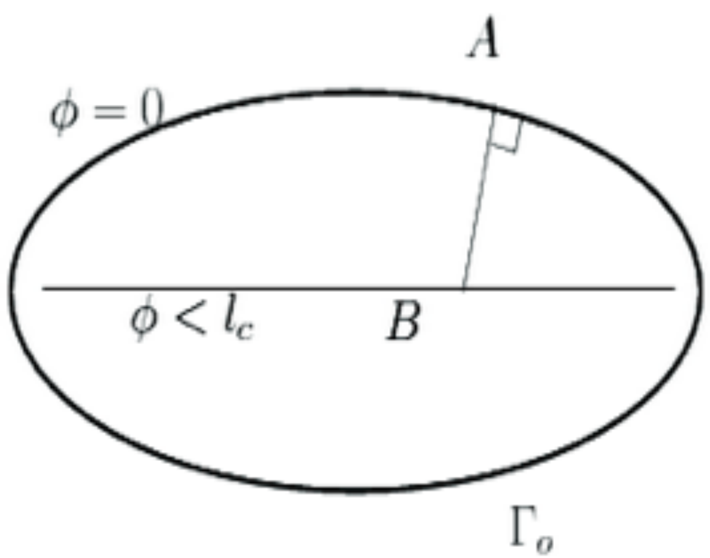


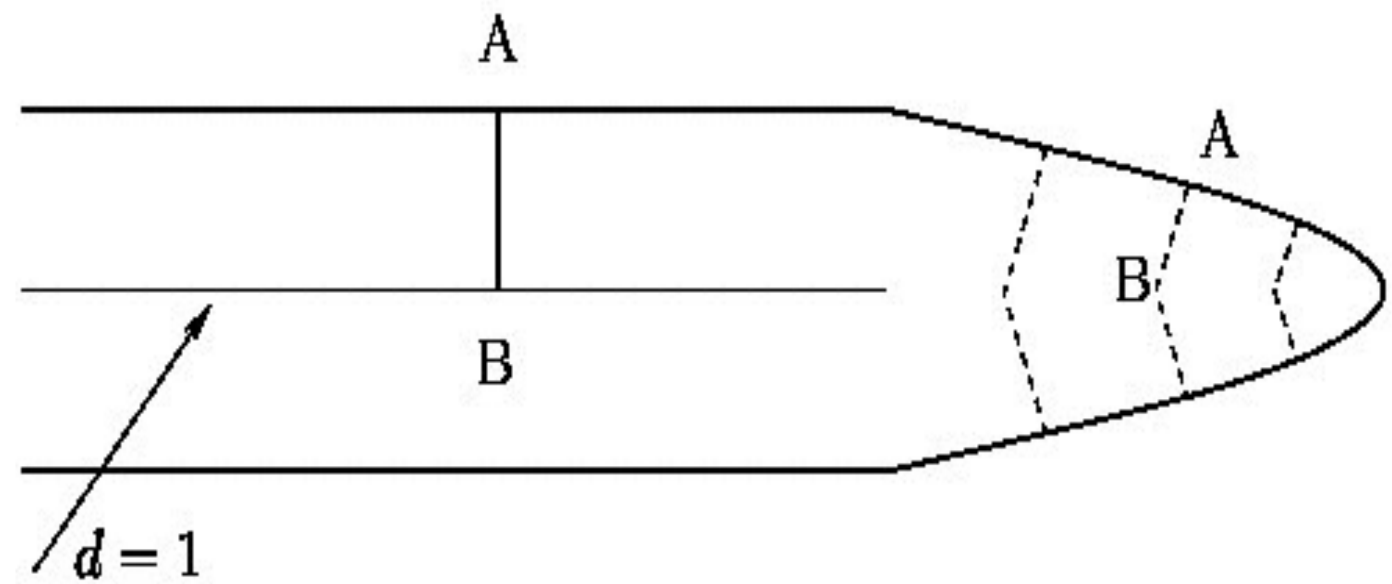
$$\text{On } [b,c] : \overline{y} = \frac{\int_b^c y d'(\phi)\,\mathrm{d}x}{\int_b^c d'(\phi)\,\mathrm{d}x}, \quad \overline{\dot{d}}(\boldsymbol{x}) = \frac{\int_b^c \dot{d}\,\mathrm{d}x}{\int_b^c \,\mathrm{d}x}$$

$$\text{On } [c,d] : \overline{y} = \frac{\int_c^d y d'(\phi)\,\mathrm{d}x}{\int_c^d d'(\phi)\,\mathrm{d}x}, \quad \overline{\dot{d}}(\boldsymbol{x}) = \frac{\int_c^d \dot{d}\,\mathrm{d}x}{\int_c^d \,\mathrm{d}x}$$

Non-local driving force along the gradient of damage

$$\overline{Y}(AB) = \frac{\int_{AB} Y\, d'(\phi)(1-\frac{\phi}{\rho_0})\, \mathrm{d}\phi}{\int_{AB} d'(\phi)(1-\frac{\phi}{\rho_0})\, \mathrm{d}\phi}$$



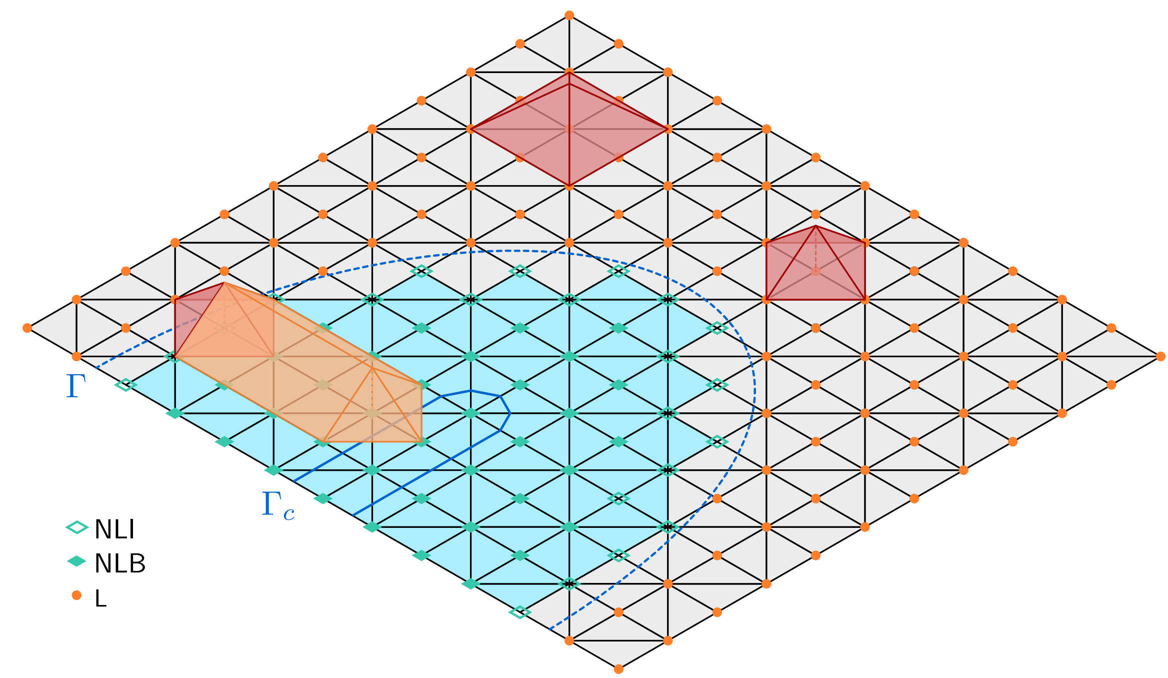


Continuous transition from local to non-local since the length over which average is performed rises from 0 to lc.

Implementation aspects (I)

Damage update

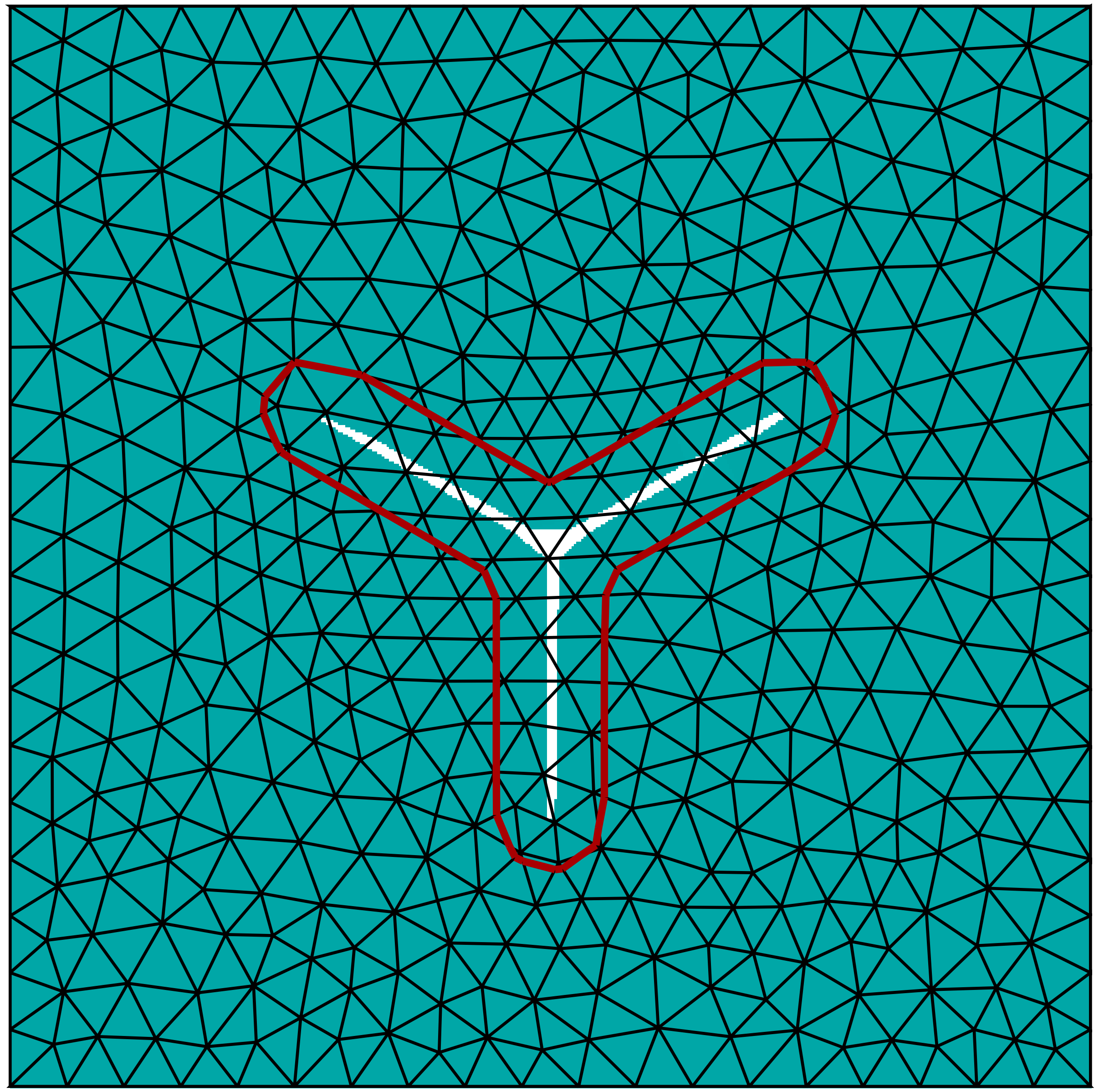
- Damage is a nodal quantity (as in damage gradient models)
- Damage update is local at the node in the local zone
- Damage gradient is monitored in eache element
- Damage update ties nodes in the non-local zone. The tie is made by fast marching

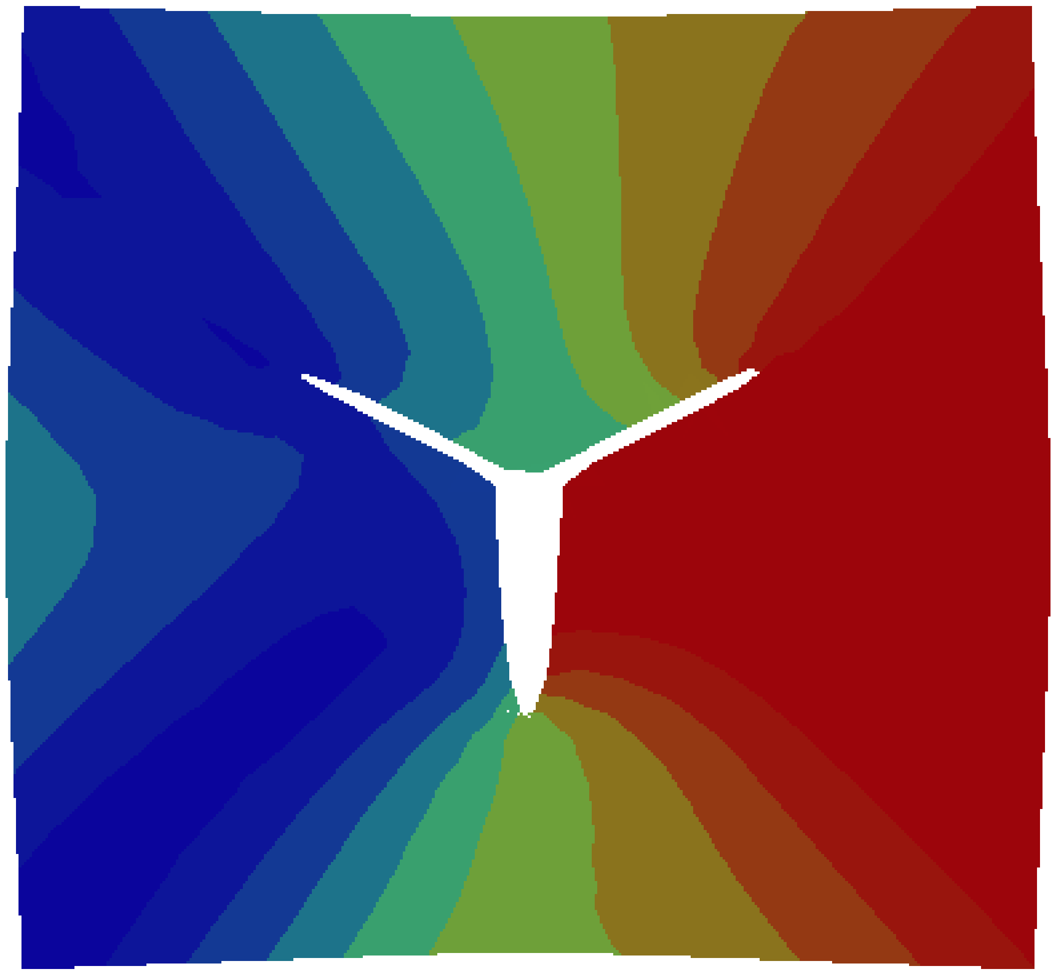


CCL: NO matrix solve for damage update with TLS
This is an important advantage in quasi-static analysis and ESSENTIAL for explicit dynamics analysis

Implementation aspects (II)

X-FEM enrichment to introduce displacement jumps





Displacement

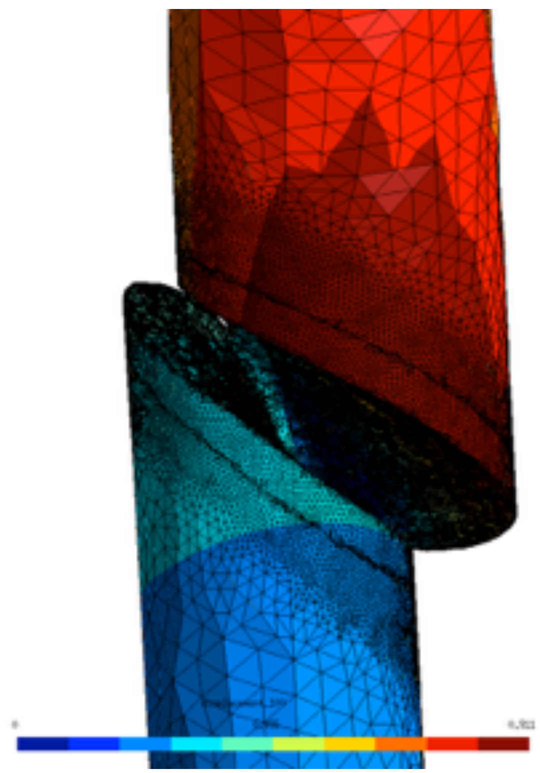
Implementation aspects (III)

Capturing length scale

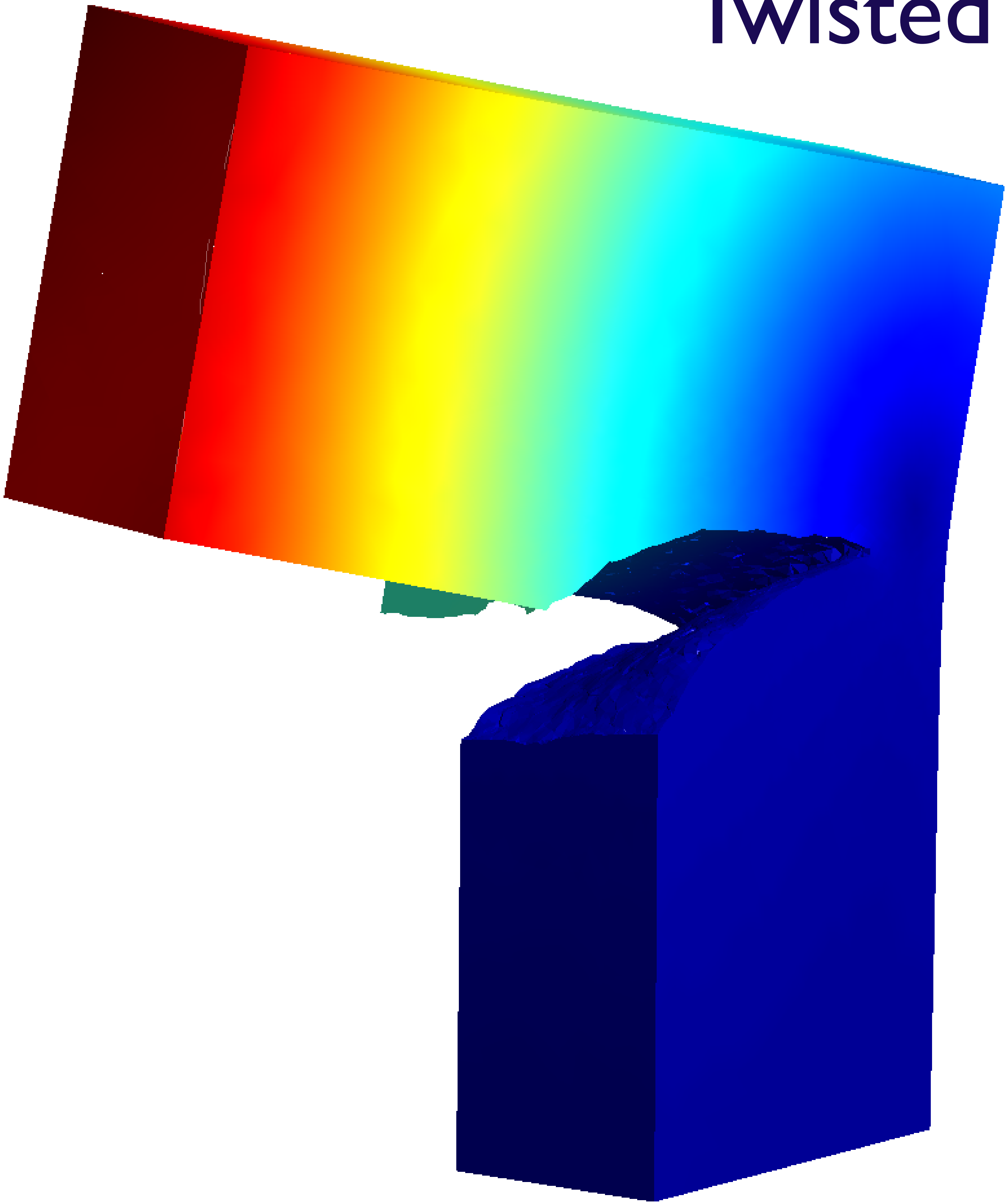
- It takes small meshes to capture the localization length (say 5 elements per lc).
- Thanks to X-FEM mesh may be derefined away from moving tips.
- Local-Global solver (from A. Duarte et al.) is on the way: goal get TLS simulation time <= 10 times LEFM analysis. Duarte Talk : workshop 1 just after coffee

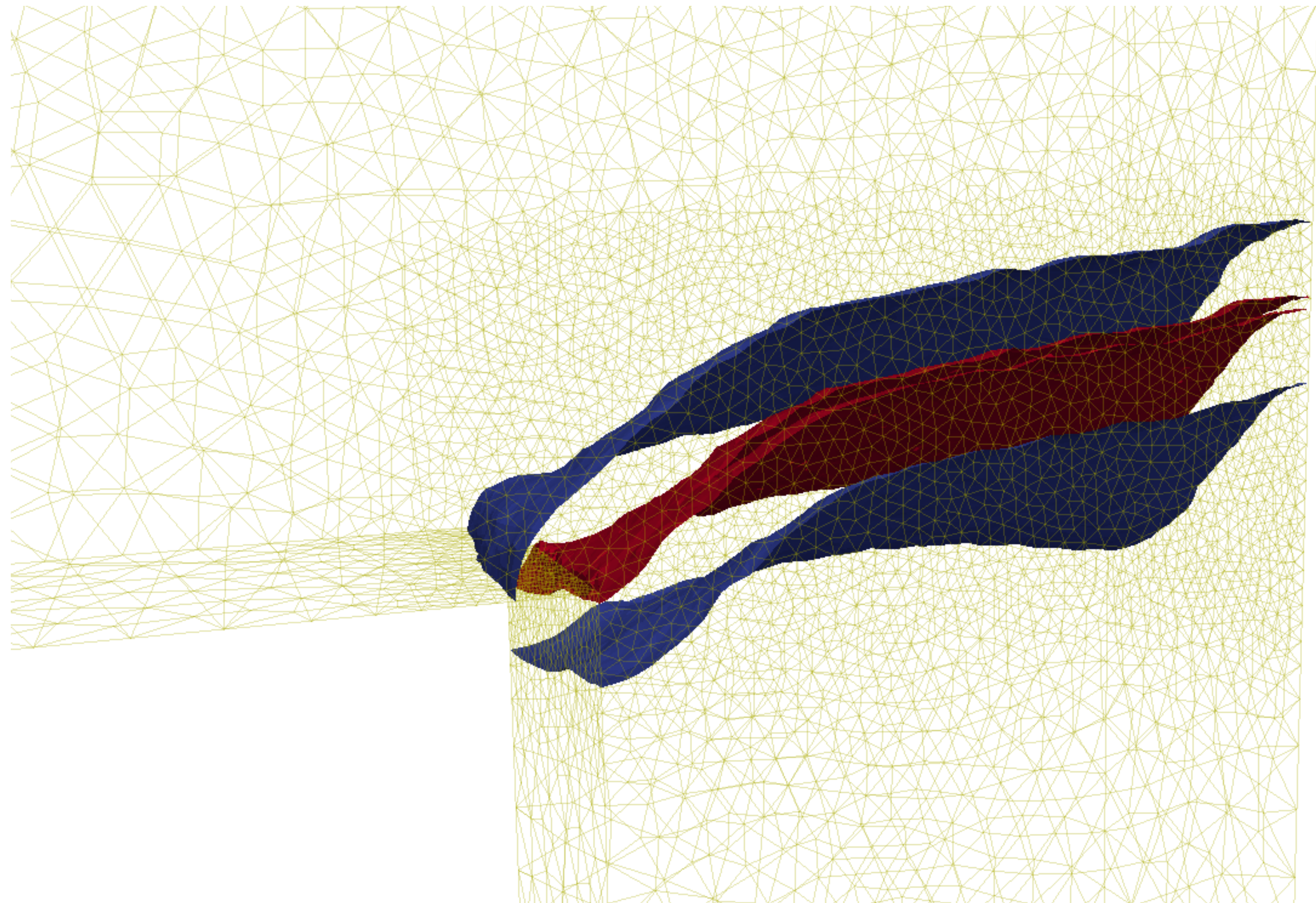
Some 3D numerical experiments

Chalk twist

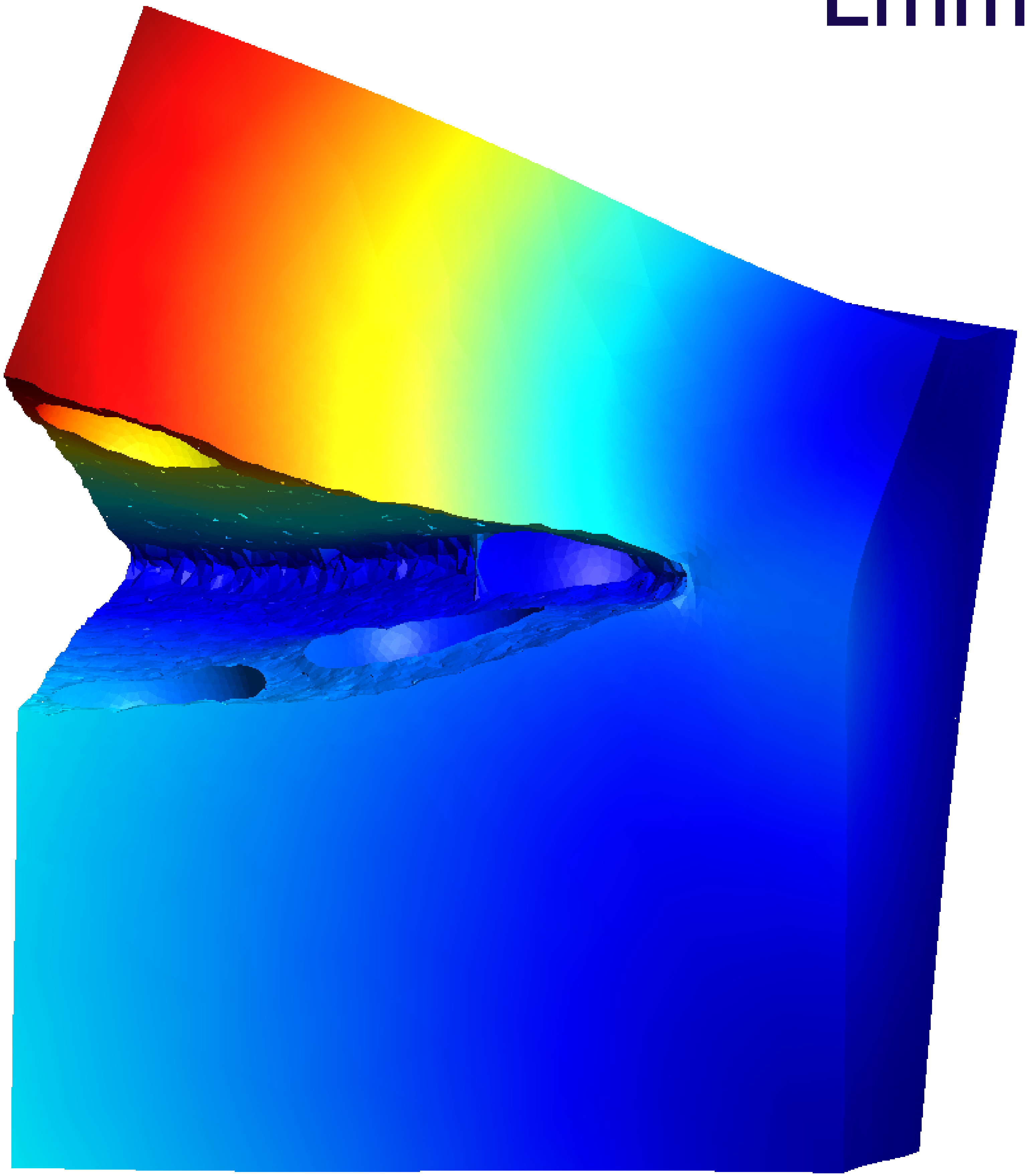


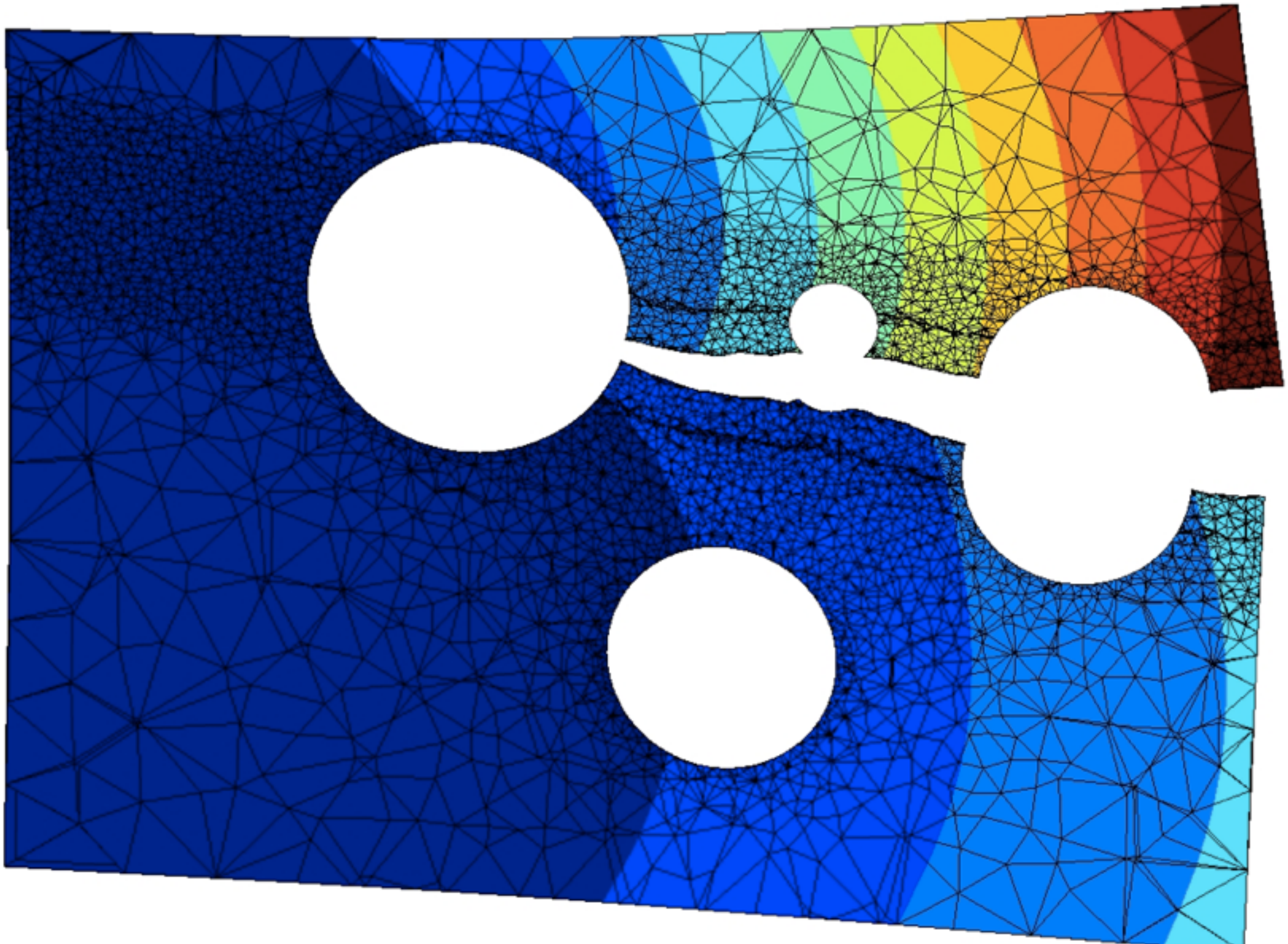
Twisted L-shape

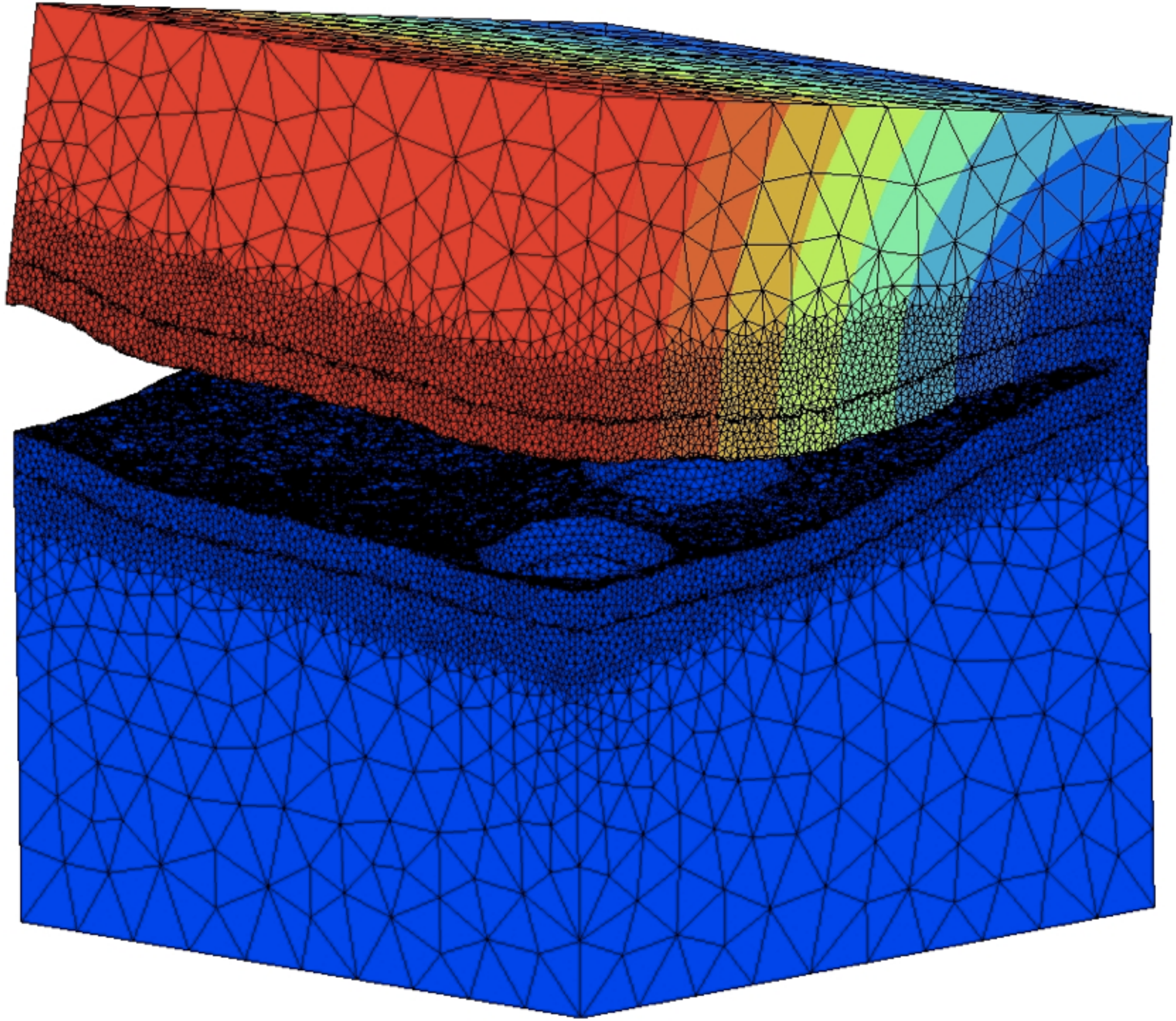




Emmental







displacement 340

A step toward debonding

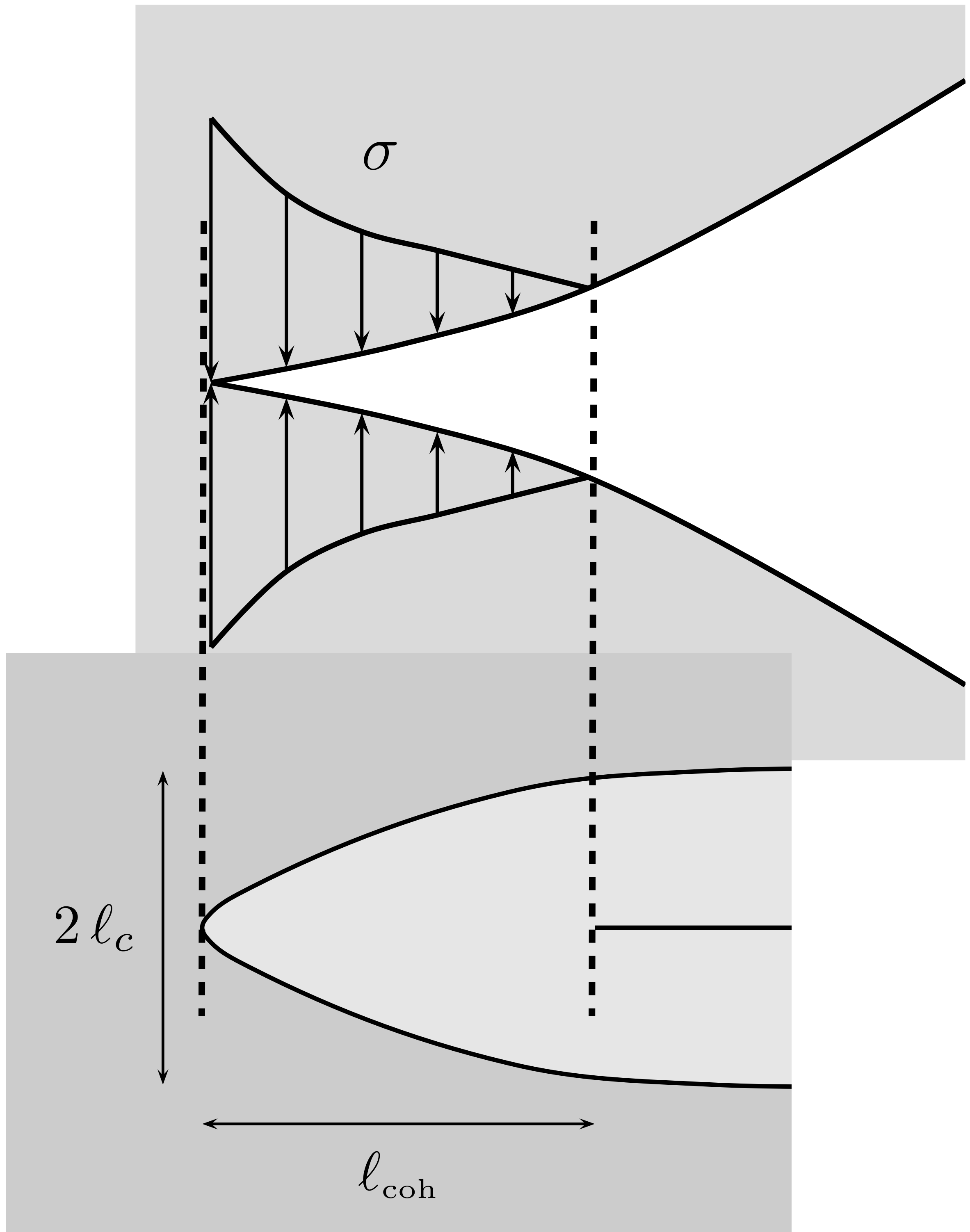
Relationship between
Cohesive Zone Model
and
Thick Level Set models

Is TLS indeed a larger set than CZM ?

Geometrical and mechanical similarities

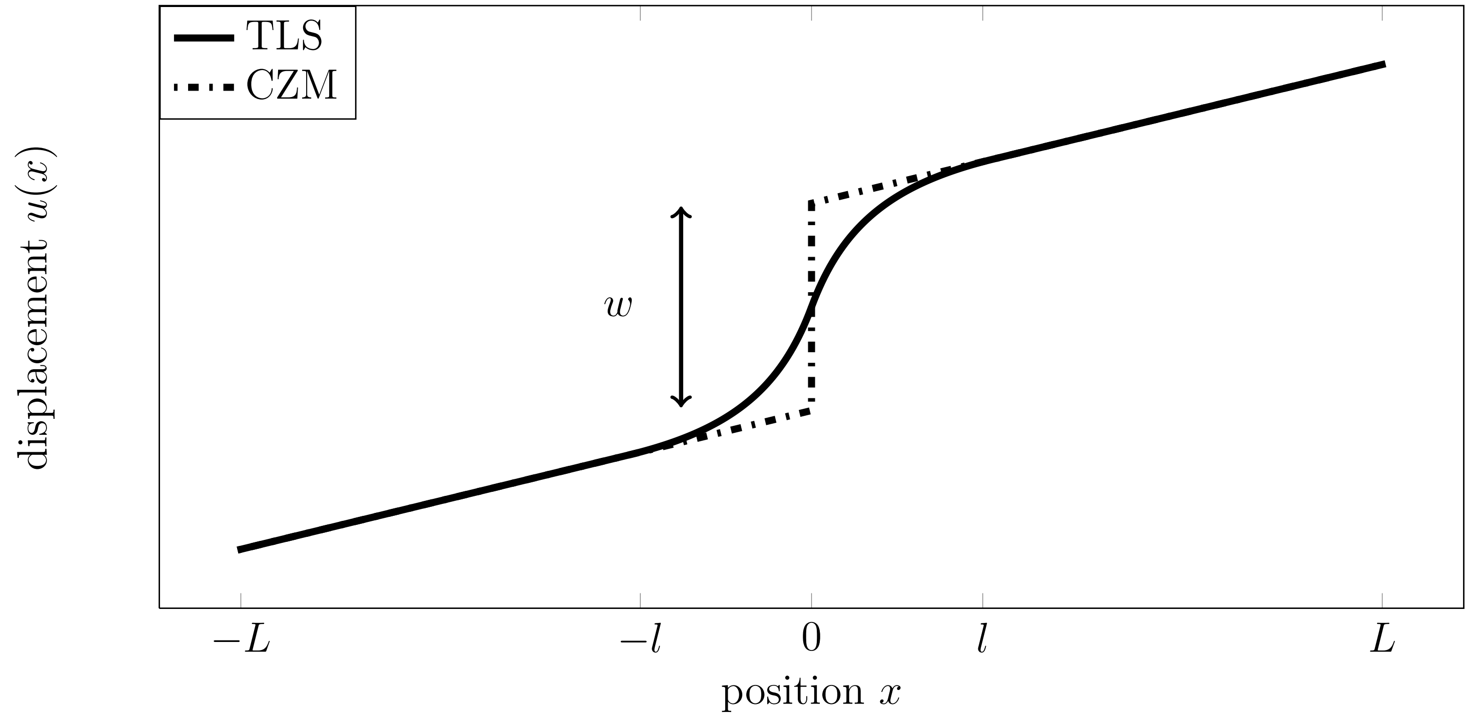
CZM

TLS



May we preserve
lcoh as lc -> 0 ?

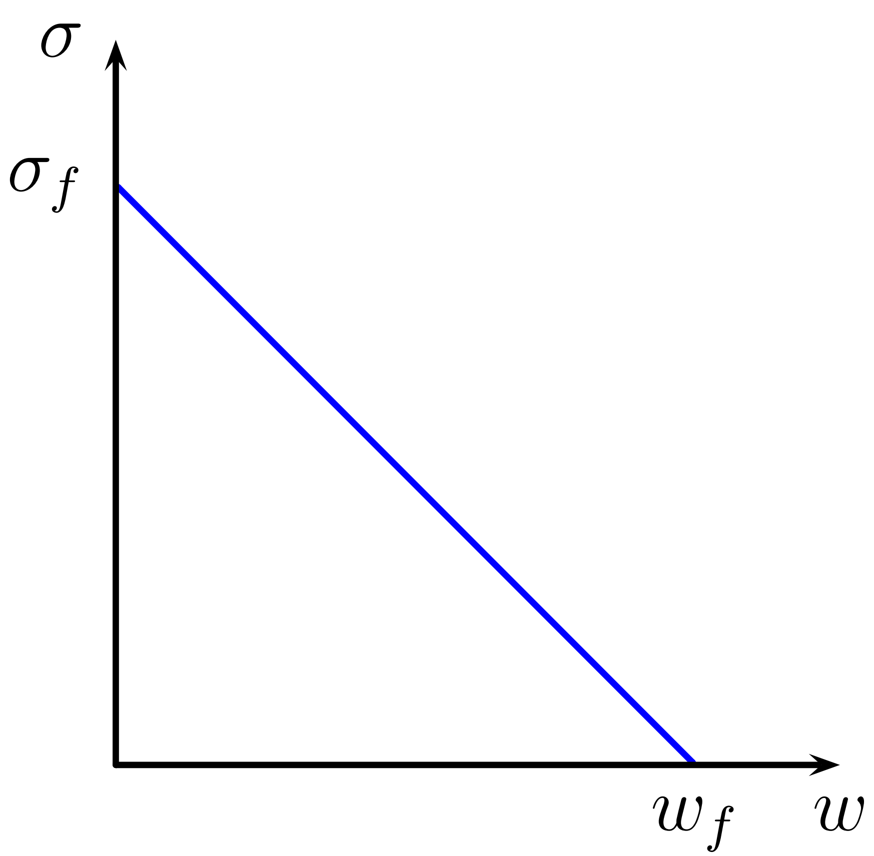
CZM and TLS 1D equivalence



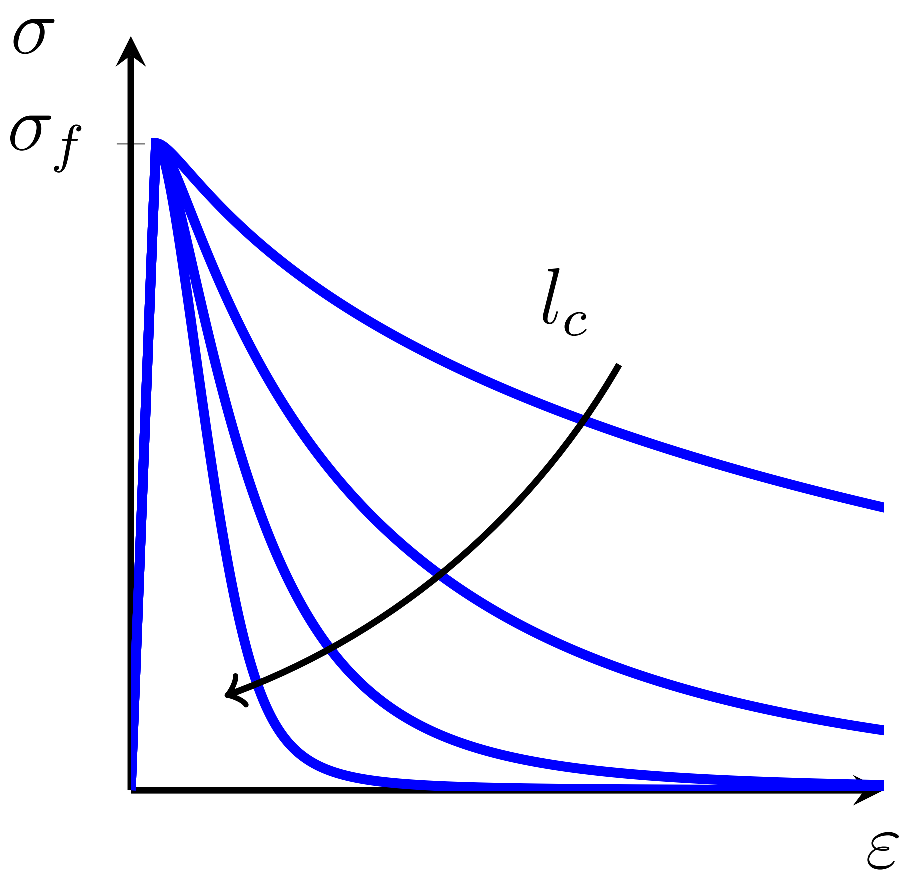
For any given stress, we impose same energy, dissipation and elongation in both models.

Note that the analysis was already carried out with other non-local approach (Cazes et al 2009, Lorentz et al. 2012)

From CZM to TLS

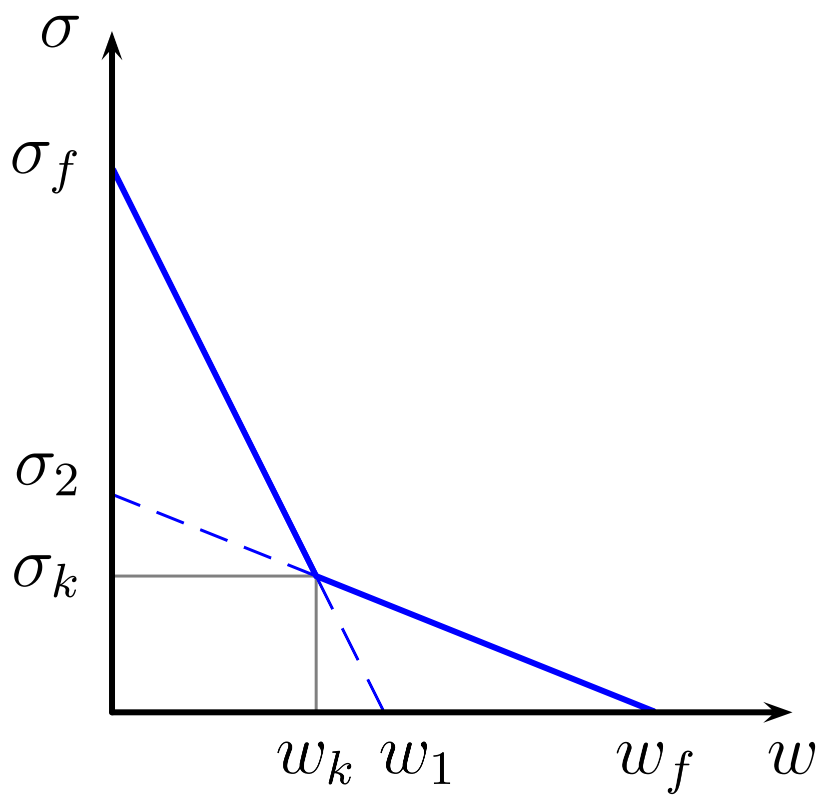


(a) Cohesive linear law.

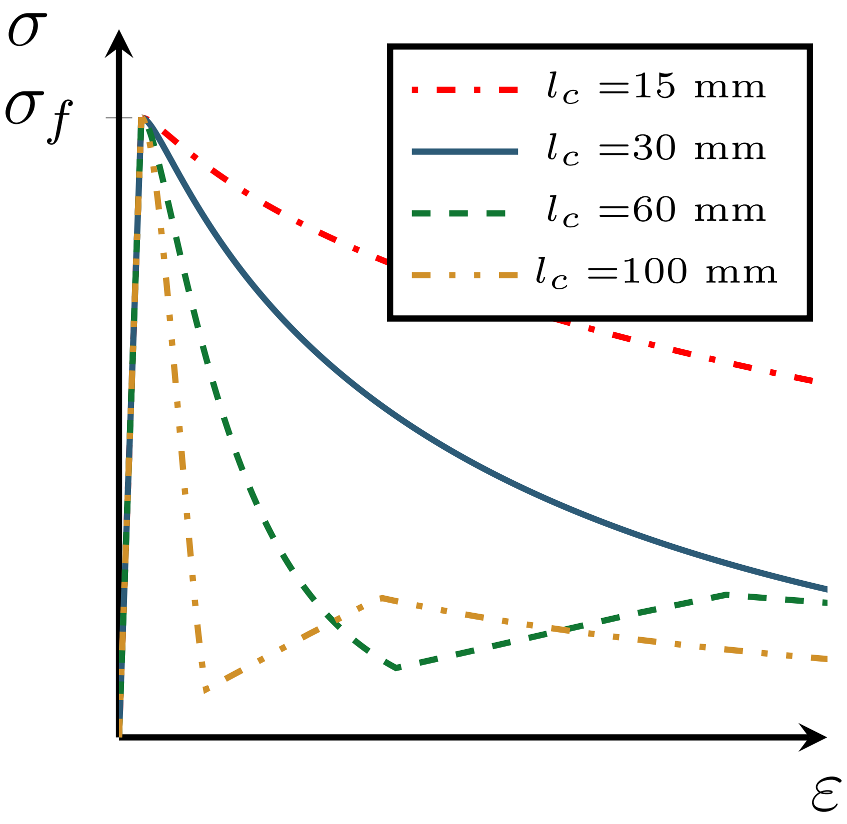


(b) TLS equivalent local behavior for different ℓ_c values. Increasing values of ℓ_c are indicated by the arrow.

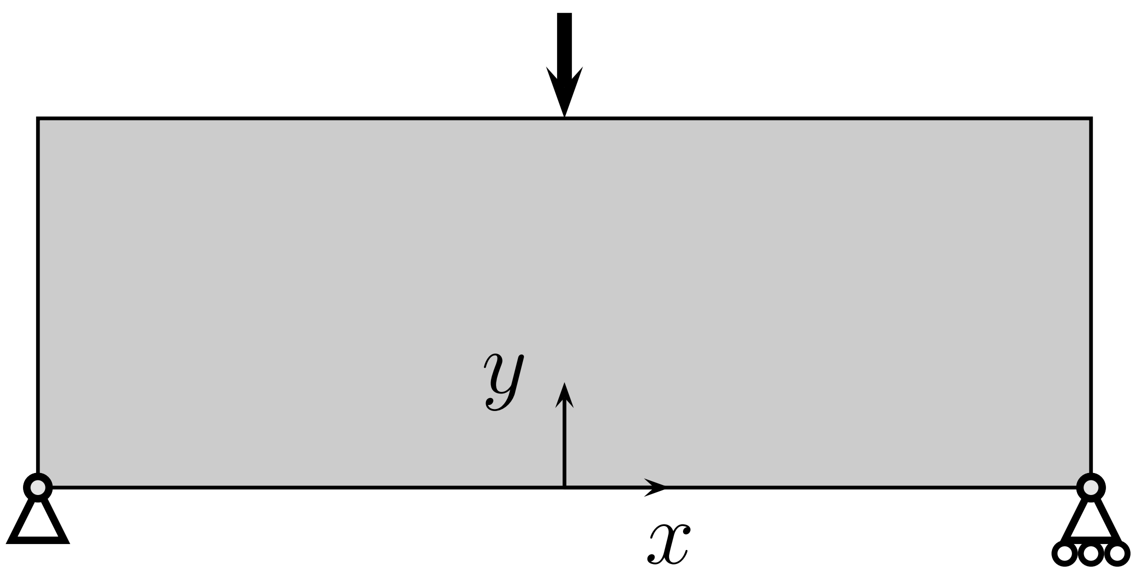
Bi-linear cohesive law case

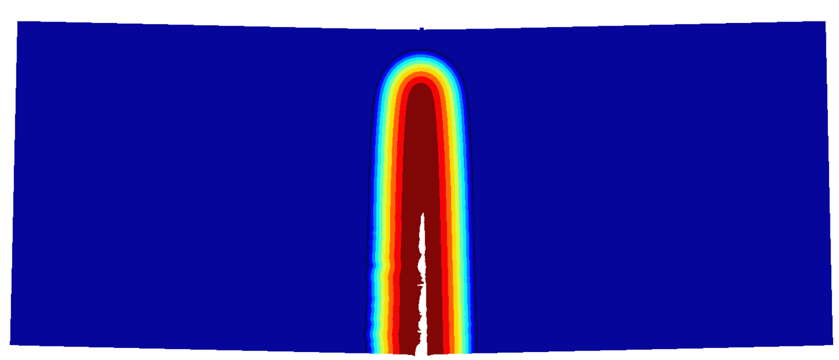


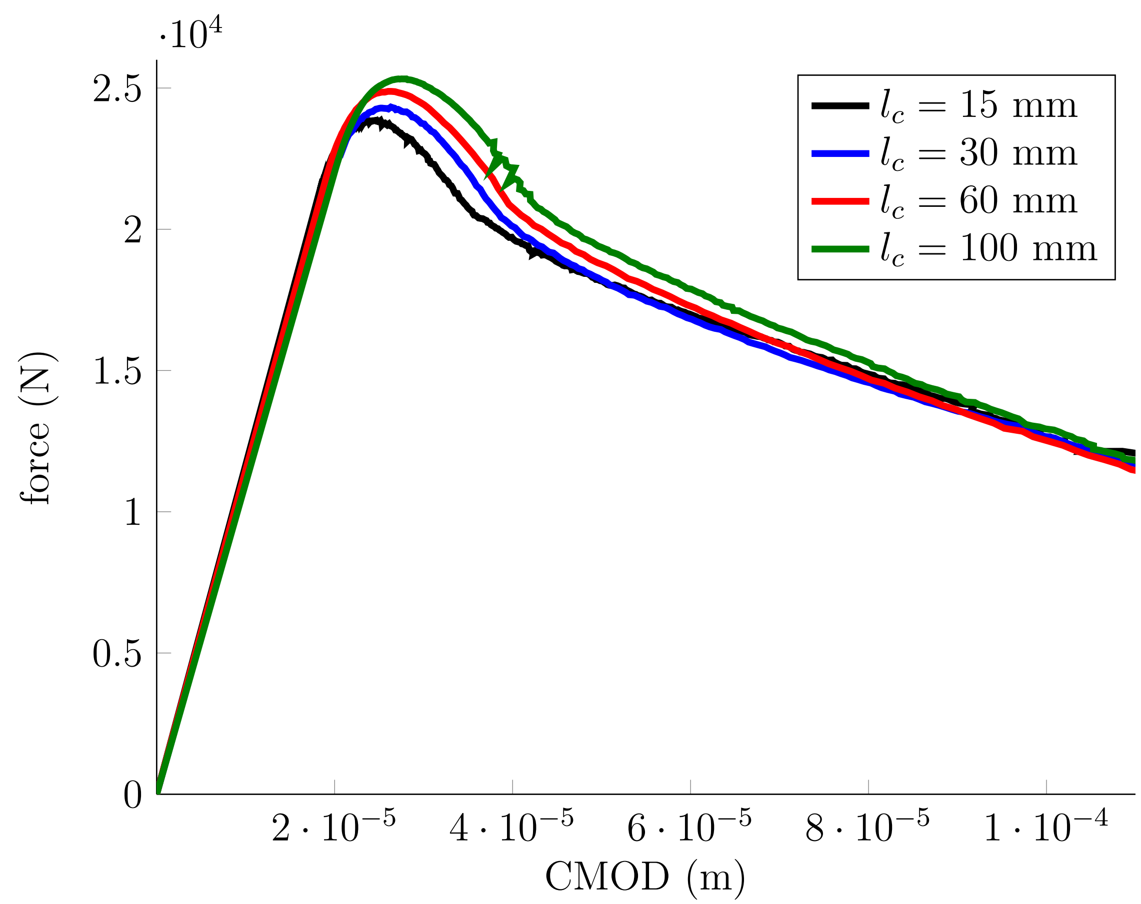
(a) Cohesive bi-linear law.



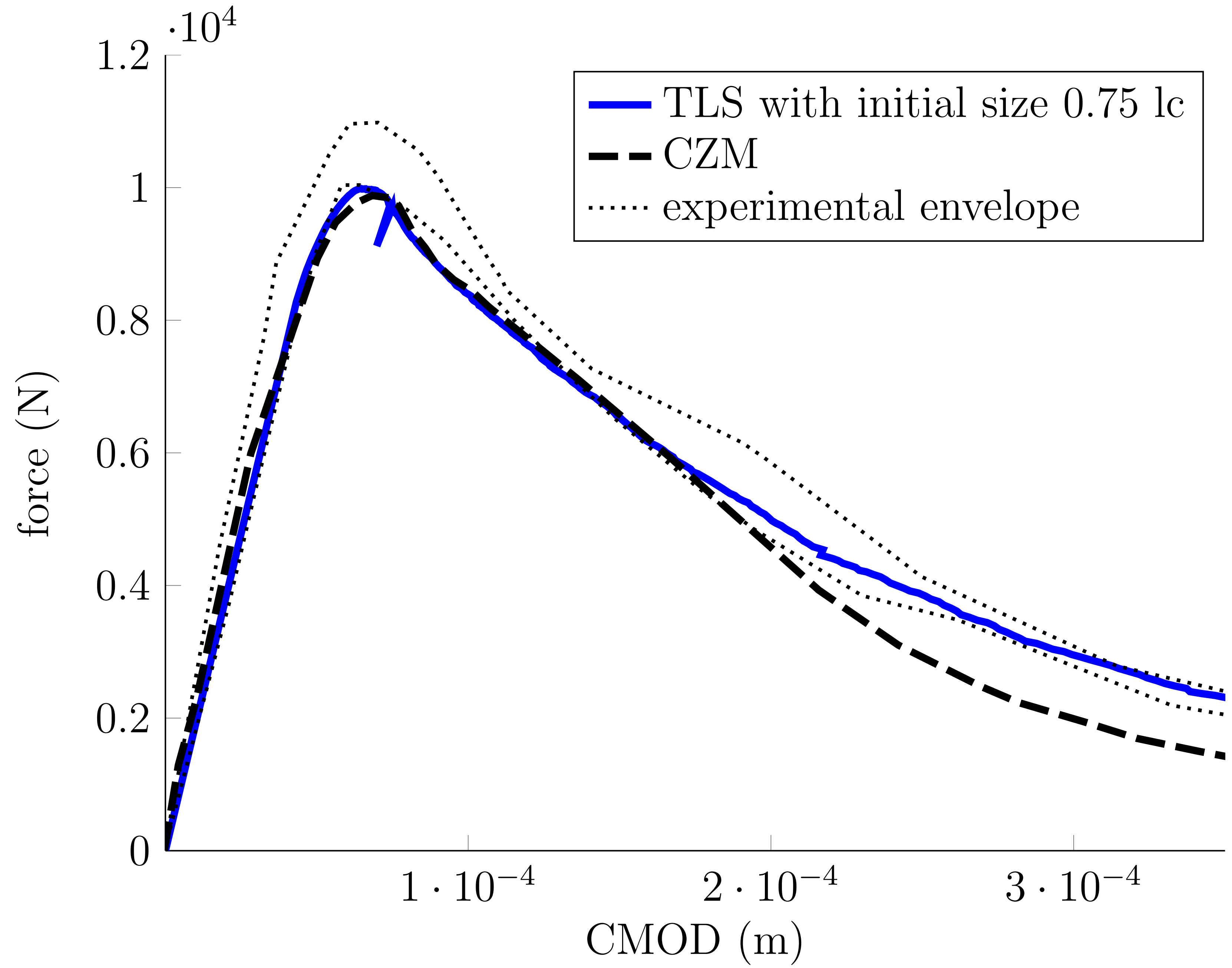
(b) TLS equivalent local behavior for different ℓ_c values.


y
x

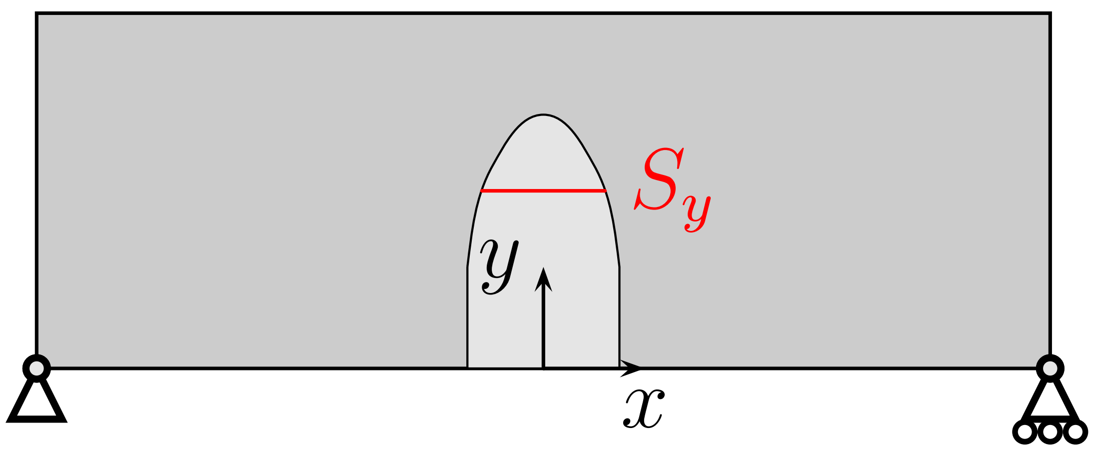



$\cdot 10^4$
2.5
2
1.5
1
0.5
0
force (N)
$2 \cdot 10^{-5}$
$4 \cdot 10^{-5}$
$6 \cdot 10^{-5}$
$8 \cdot 10^{-5}$
$1 \cdot 10^{-4}$
CMOD (m)
$l_c = 15$ mm
$l_c = 30$ mm
$l_c = 60$ mm
$l_c = 100$ mm

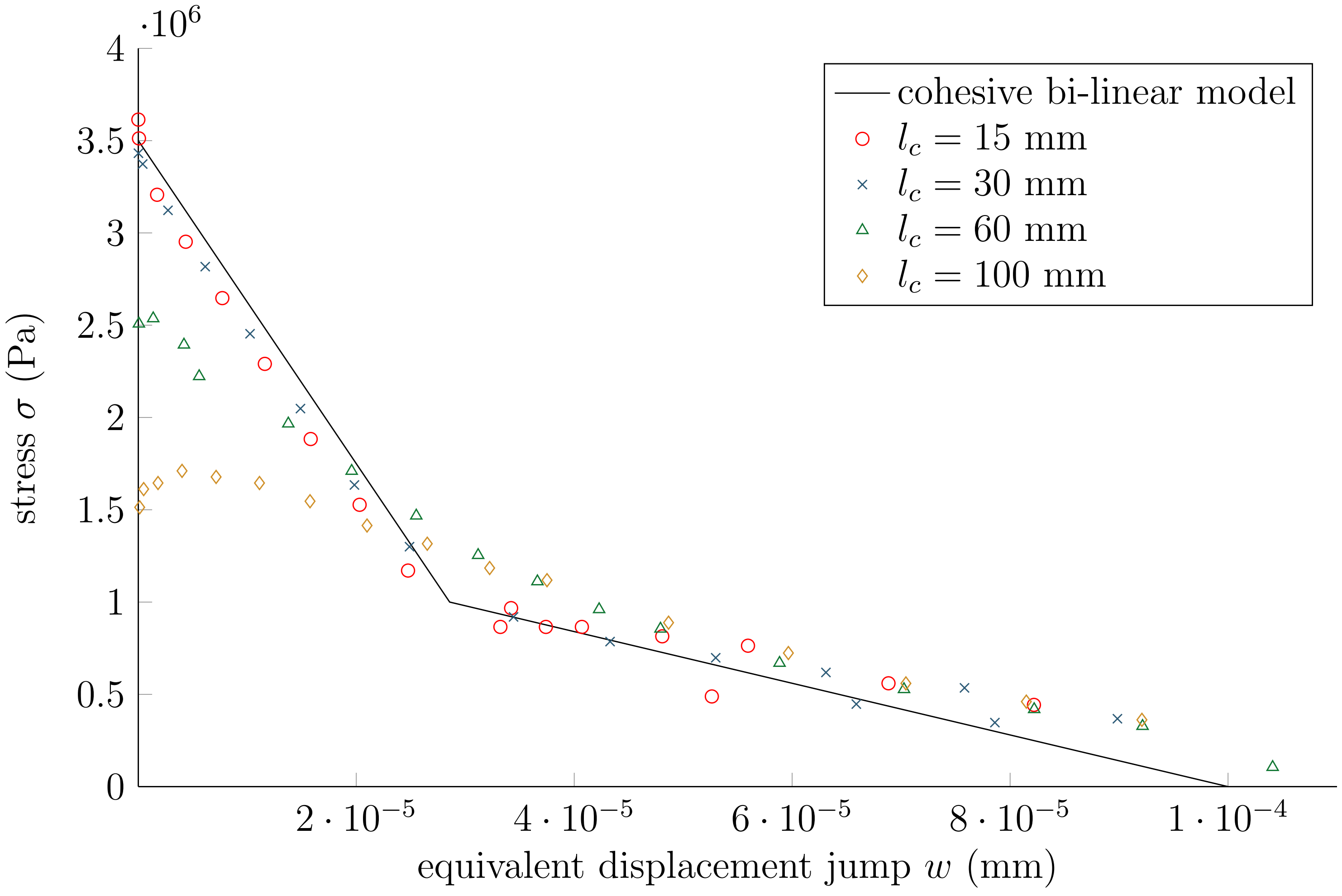
Force - CMOD results



(b) Load-CMOD curve. Plain curve is TLS (raw data), dashed one is CZM and dotted ones are the experimental envelope.

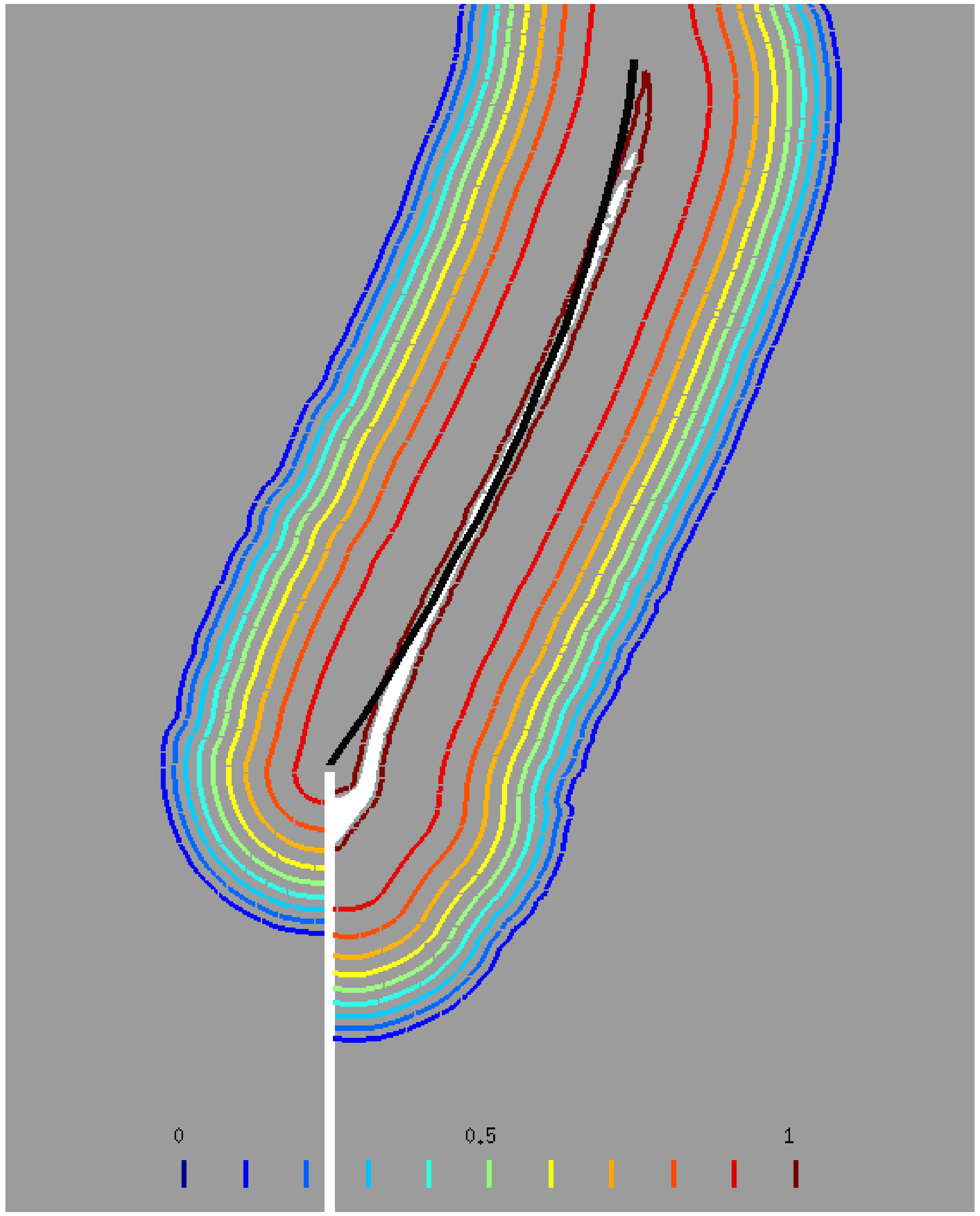


S_y
y
x



$\cdot 10^6$
4
3.5
3
2.5
2
1.5
1
0.5
0
stress σ (Pa)
$2 \cdot 10^{-5}$
$4 \cdot 10^{-5}$
$6 \cdot 10^{-5}$
$8 \cdot 10^{-5}$
$1 \cdot 10^{-4}$
equivalent displacement jump w (mm)
cohesive bi-linear model
$l_c = 15$ mm
$l_c = 30$ mm
$l_c = 60$ mm
$l_c = 100$ mm

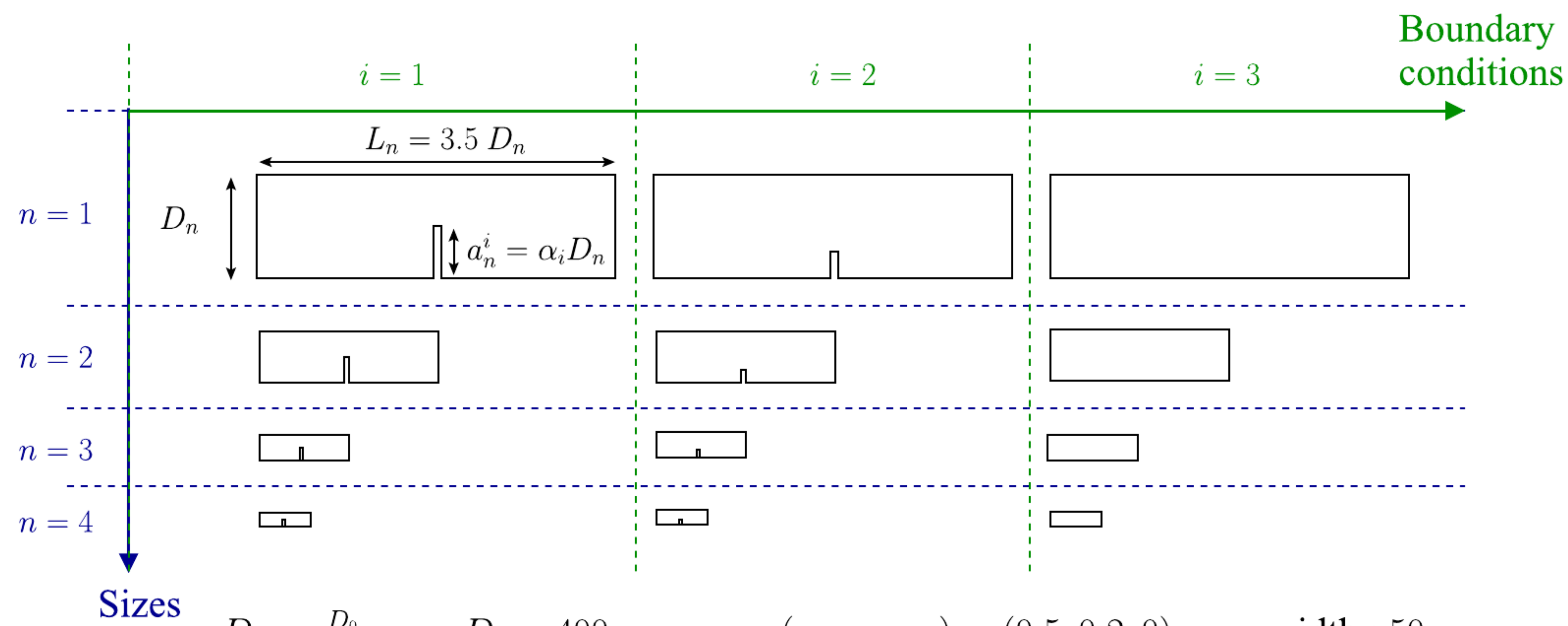
Comparison between TLS and CZM path



Analysis of size and shape effects in concrete beams

Work in progress collaboration
with D. Gregoire and G. Pijaudier-Cabot

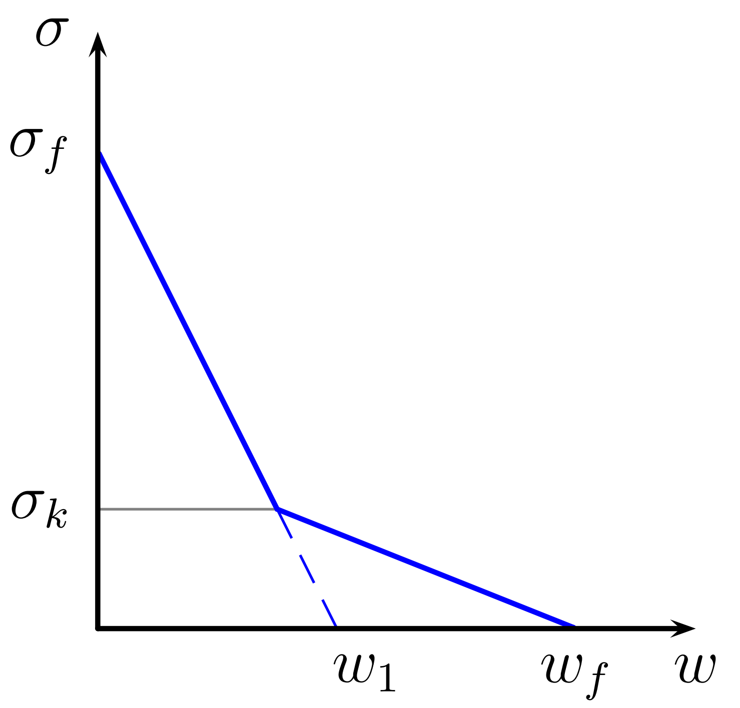
Size Effect experiments on concrete beams (three point bending)

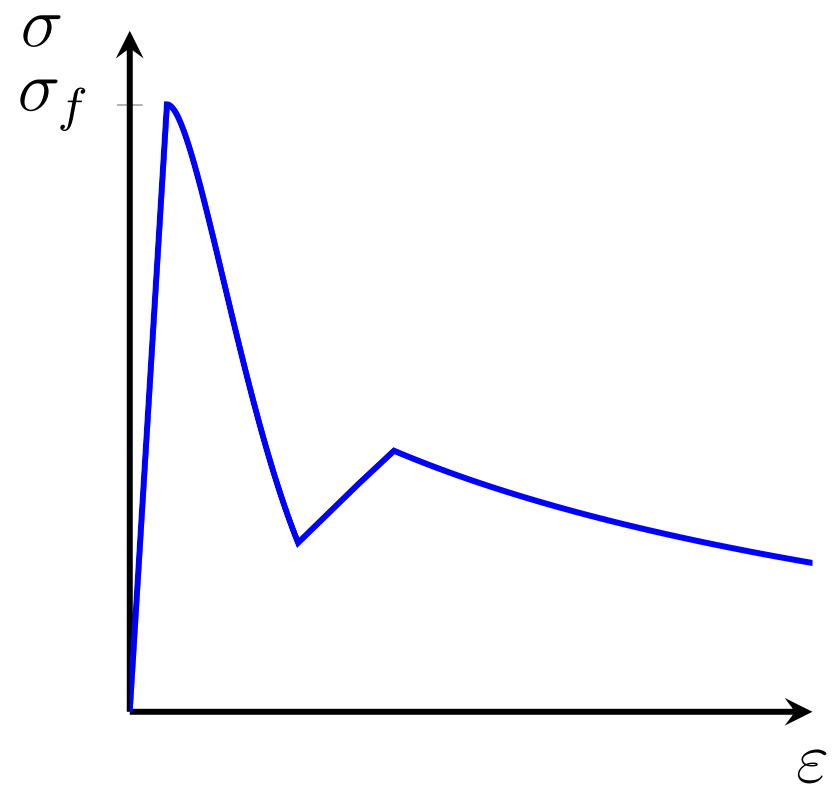


$D_n = \frac{D_0}{2^{n-1}}$; $D_0 = 400\ mm$; $(\alpha_1, \alpha_2, \alpha_3) = (0.5, 0.2, 0)$; width : $50\ mm$

D. Grégoire, L. Rojas-Solano, and G. Pijaudier-Cabot, "Failure and size effect for notched and unnotched concrete beams," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 37, no. 10, pp. 1434–1452, 2013.

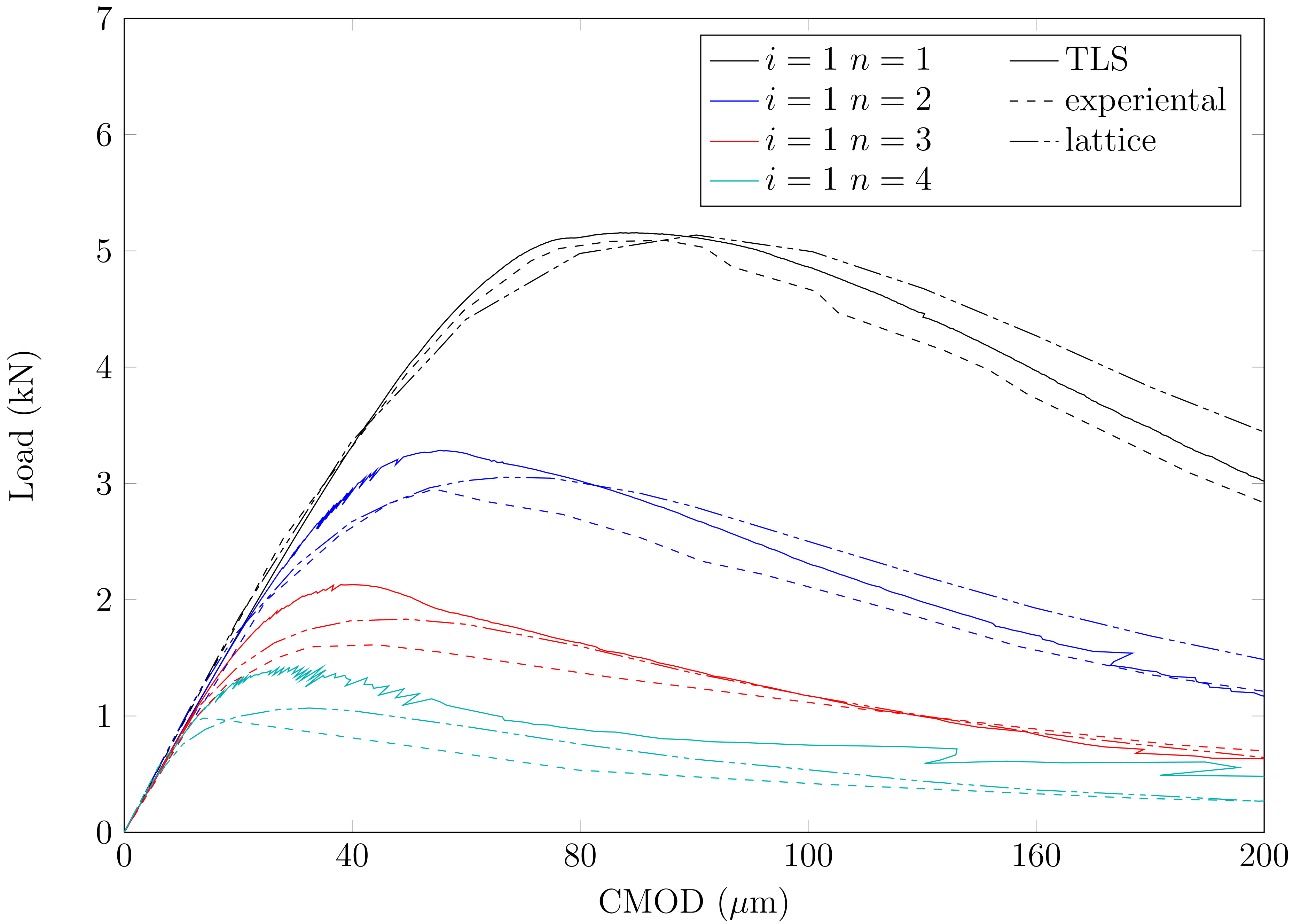
Identified TLS parameters





	TLS simulation parameters
σ_f	3.8 MPa
σ_k	1.8 MPa
w_1	23 μm
w_f	55 μm

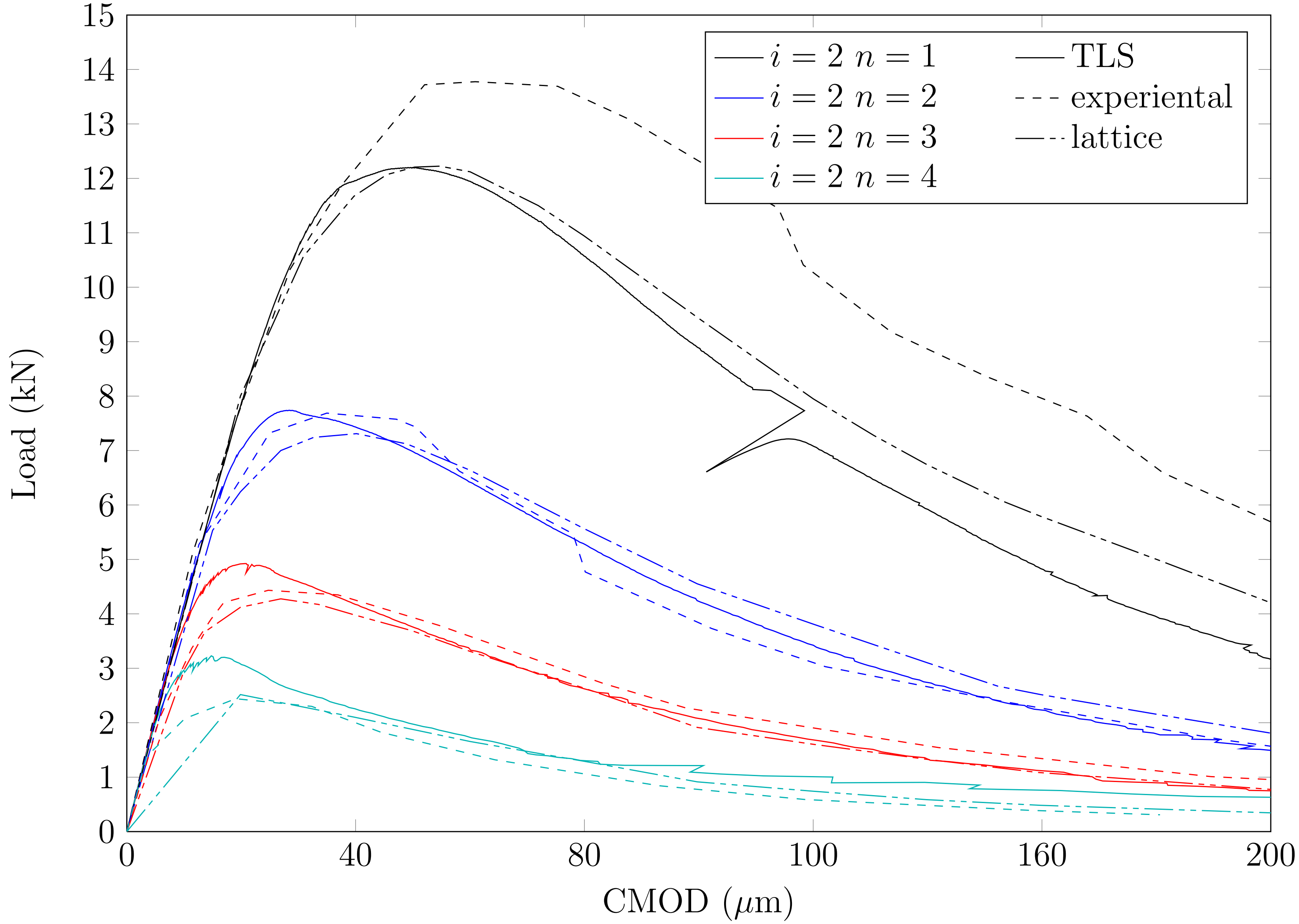
Deep notch



Lattice model:

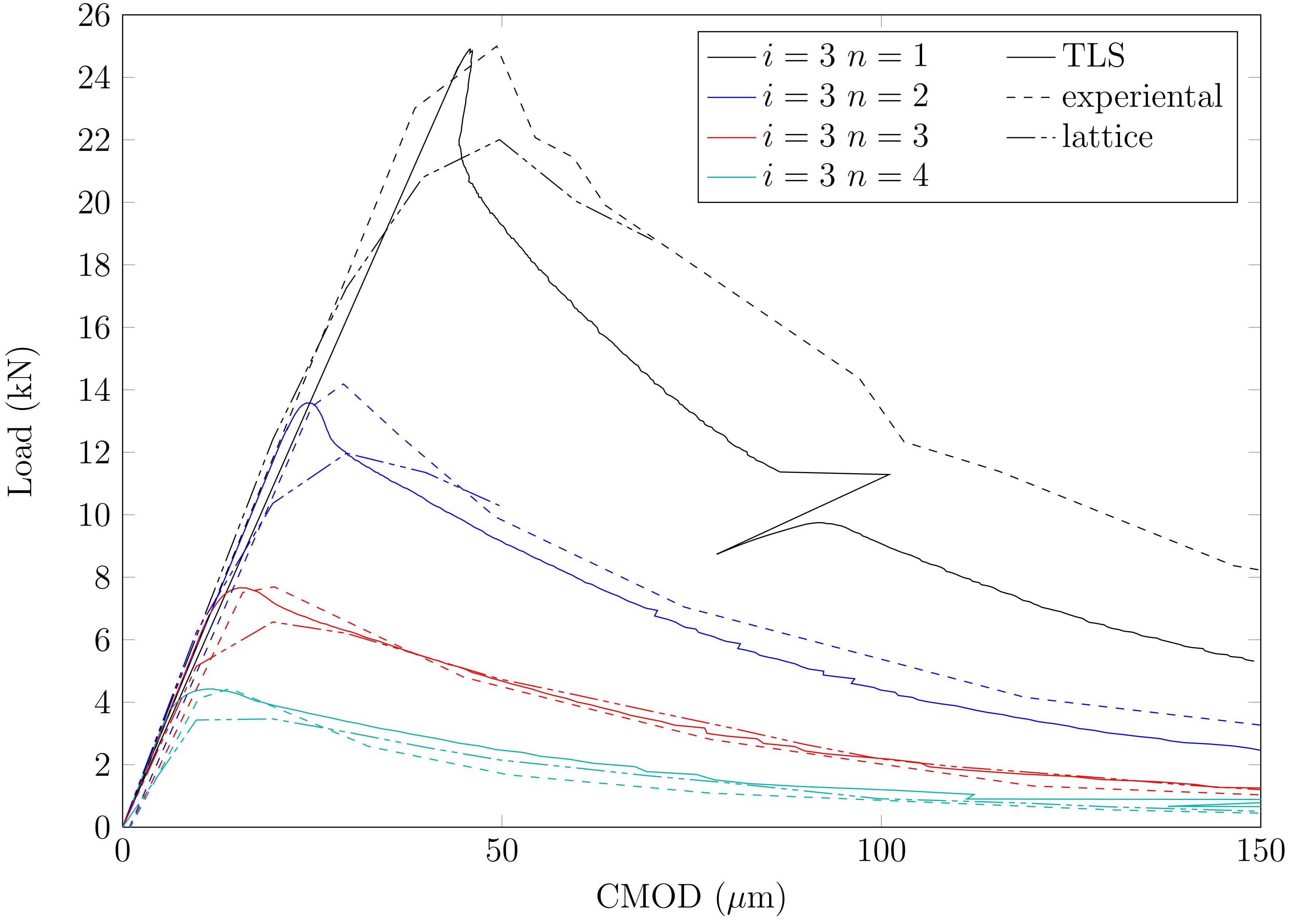
P. Grassl, D. Grégoire, B. Rojas-Solano, Laura, and G. Pijaudier-Cabot, "Meso-scale modelling of the size effect on the fracture process zone of concrete," *International Journal of Solids and Structures*, vol. 49, no. 13, pp. 1818–1827, 2012.

Small Notch



15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
Load (kN)
0
40
80
100
160
200
CMOD (μm)
i = 2 n = 1
i = 2 n = 2
i = 2 n = 3
i = 2 n = 4
TLS
experiental
lattice

No notch



26
24
22
20
18
16
14
12
10
8
6
4
2
0
Load (kN)
0
50
100
150
CMOD (μm)
i = 3 n = 1
i = 3 n = 2
i = 3 n = 3
i = 3 n = 4
TLS
experiental
lattice

Conclusions on TLS

- Crack appears automatically and X-FEM may be used to model displacement jump
- Automatic seperation between local and non-local zone
- Smooth transition into non-locality
- No matrix solve for damage update
- For quasi-brittle material, TLS generalizes CZM giving thickness 2lc
- It seems to fit size effect for concrete beams

Under way

- From visco-damage to fracture (with Ifsttar O. Chupin and J-M Piau) poster 2pm
- Several damage variables (with C. Comi Politecnico de Milano) and damage anisotropy
- Cracks in reinforced concrete
- Fragmentation (with J. Dolbow & A. Sterchic Duke University)
- Ductile failure
- CFRAC conference 14-16 June 2017 Nantes